Soft or Hard Connections of Beam and Shell Elements in FE Models?

H. Werkle[1]

Keywords: Finite Element Modeling, Beam, Plate, Deep Beam, Column

Abstract

The connection of beam elements with shell elements in finite element models can be based on the assumption of a linear distribution of the displacements or on a linear distribution of the stresses. The first implies rigid body kinematics and is called "hard" connection whereas the second is based on equilibrium conditions and is called "soft" connection. The paper describes the formulation for the "soft connection" and its advantages over the widely used "hard connection". This approach has first been applied to the column-slab problem in the analysis of flat slabs. As examples for the application of the approach the beam-wall problem is considered. It demonstrates the practical value of the approach.

1. Introduction

In structural mechanics different stress descriptions are used for plates and beams. The most general description is the stress tensor used for three-dimensional solids or plates in plane stress. For beams and plates in bending, stress resultants like bending moments and shear forces are very successful engineering concepts. Sometimes it is efficient to use both stress descriptions in a finite element model. However the basic differences in both models may result in problems in such models. A point force e.g. which is a successful concept in beam models causes stress and displacement singularities in plane stress models.

There are many cases where the transition between stresses and stress resultants has to be modelled. A typical example is the connection of beam elements with plane stress finite elements. In the beam element, the stresses are integrated to stress resultants as longitudinal forces, shear forces and bending moments. However, only distributed loads are allowed for the plate in order to avoid stress and displacement singularities. In addition the moment communication at the end of the beam element has to be modelled, Fig.1. Hence, the consistent modelling of the connection of a beam element with plane stress elements is not obvious. Another typical problem in reinforced concrete structures is the connection of columns of flat slabs with the slab where the columns are represented by beam elements and the slab by plate elements in bending.

2. Models for element transitions

There are various ways to model transitions between different element types. These are

- Multipoint constraints
    - Transformation method
    - Lagrange multiplier method
    - Penalty method
- Transition elements
- Engineering approaches
- Equivalent stiffness transformation (EST)

[1]Professor Dr.-Ing., University of Applied Sciences Konstanz, Germany, E-mail: werkle@fh-konstanz.de

Paper presented at the International Conference "Performance based Engineering for 21st Century", TU Iasi, Romania, 2004

For multipoint constraints it is assumed that the displacements of the nodes can be described by rigid elements. For the beam-plate connection in Fig. 1(a), e.g., these rigid constraints can be described for the nodes 1, 2 and 3 as

$$u_1 = u_2 + \varphi_2 \cdot h/2, \quad u_3 = u_2 - \varphi_2 \cdot h/2, \quad v_1 = v_2, \quad v_3 = v_2.$$

The degrees of freedom of the 'slave' nodes 1 and 3 are expressed by the translations and the rotation of 'master' node 2. This method is called the transformation method. Other methods to take into account multipoint constraints are the Lagrange multiplier method and the Penalty method.

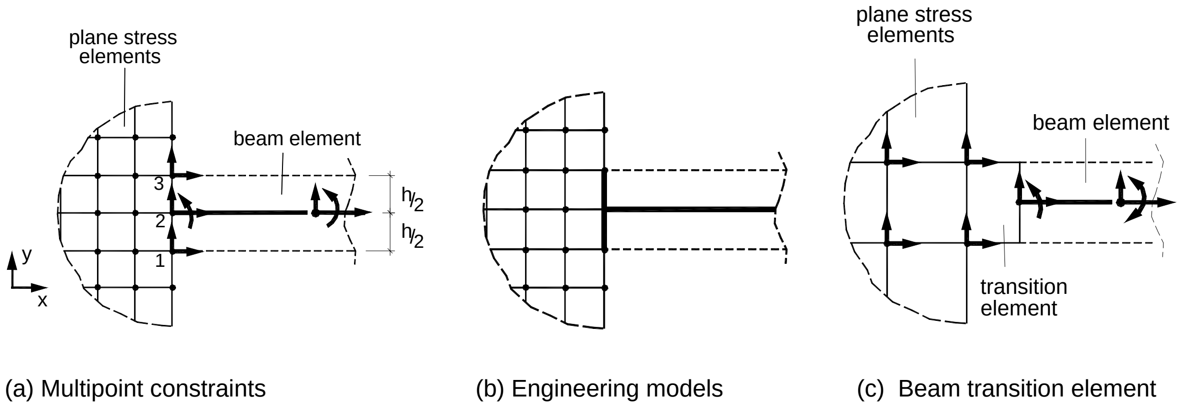

Figure 1: Models for element transitions

Transition elements are another approach to connect dissimilar finite element domains. They are special finite elements to connect two elements of different types, Fig. 1(c).

For engineering purposes, models as shown in Fig. 1(b) are used. The short beam elements act as rigid constraints. If the stiffness of the beam elements chosen is large enough, the model fulfils approximately the multipoint constraint conditions.

A new approach for element transitions is developed based on the fulfillment of the equilibrium conditions and the approximate compatibility of the displacements between the two elements [2-5]. It is called Equivalent Stiffness Transformation or EST. The model avoids rigid constraints, allows multiple element connections, doesn't result in stress singularities and hence is suited for adaptive meshing.

3. Equivalent stiffness transformation

In the Equivalent Stiffness Transformation two stress systems, the source system and the target system are considered. The stiffness matrix formulated in the source system is transformed by EST into the target system in which the global equations are formulated.

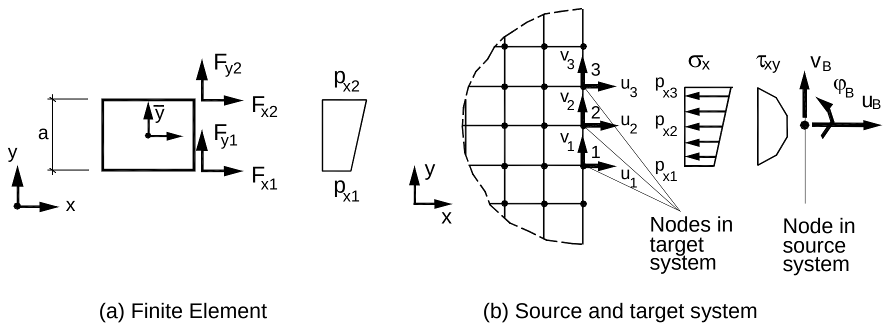

Figure 2: EST model

In the example of the beam-plane stress element connection, the source system is the linear stress distribution in the beam which can be integrated to give stress resultants. The stiffness matrix of the beam corresponds to the end forces and moments of the beam and the corresponding degrees of freedom. The target system is the plate in plane stress. Its degrees of freedom are the nodal displacements of the element. Hence the stiffness matrix of the beam element relating to the degrees of freedom u_B,, v_B and φ_B is to be transformed into the degrees of freedom u_1, v_1 , u_2, v_2, u_3 and v_3 of the target system, Fig. 2.

- **Method of transformation**

The transformation of the stiffness matrix in the source system is done in three steps:

Step 1: Determination of the stresses of the source system at the nodes of the target system

In the source system the stresses are expressed by the nodal forces $\underline{F}_S$. The stresses $\underline{p}_T$ at the nodal points of the target system are

$$\underline{p}_T = \underline{X} \cdot \underline{F}_S \,. \tag{1}$$

Step 2: Determination of nodal forces in the target system for the stress pattern according to step 1

The stresses are now applied as “distributed loads” to the elements of the target system. The nodal forces corresponding to these element stresses are

$$\underline{F}_T = \underline{A} \cdot \underline{p}_T \tag{2}$$

The Matrix $\underline{A}$ is obtained by assembling element matrices $\underline{A}^{(el)}$ of all elements connected to the source system. The element assemblage procedure is the same as for element stiffness matrices. The matrix $\underline{A}$ depends on the finite element type in the target system as well as on the function describing the stress variation in the source system.

Step 3: Transformation of the stiffness matrix of the source system

The transformation matrix for the element forces can be obtained easily with (1) and (2) as

$$\underline{F}_T = \underline{T}^T \cdot \underline{F}_S \tag{3}$$

where

$$\underline{T}^T = \underline{A} \cdot \underline{X} \,. \tag{4}$$

The node displacements are u_S and u_T in the source and in the target system, respectively. It can be shown that they are also transformed with the matrix $\underline{T}$ as

$$\underline{u}_S = \underline{T} \cdot \underline{u}_T \tag{5}$$

In the source system the stiffness matrix is given by

$$\underline{K}_S \cdot \underline{u}_S = \underline{F}_S \tag{6}$$

Using (3) and (5) it can be transformed into the target system by

$$\underline{K}_T \cdot \underline{u}_T = \underline{F}_T \tag{7}$$

with $$\underline{K}_T = \underline{T}^T \cdot \underline{K}_S \cdot \underline{T} \,. \tag{8}$$

- **Example: EST for a beam element and plane stress elements**

The application of the procedure described above is done for the connection of a beam element with isoparametric plane stress elements with linear interpolation functions. The beam representing the source system has the degrees of freedom and nodal forces (see Fig. 2b) to be connected with the plate

$$\underline{u}_S = \underline{u}_a = \begin{bmatrix} u_B \\ v_B \\ \varphi_B \end{bmatrix} \qquad \underline{F}_S = \underline{F}_a = \begin{bmatrix} F_{x,B} \\ F_{y,B} \\ M_{z,B} \end{bmatrix} . \tag{9}$$

The stiffness matrix of the beam with the nodes *a* and *b* can be written

$$\begin{bmatrix} \underline{K}_{aa} & \underline{K}_{ab} \\ \underline{K}_{ba} & \underline{K}_{bb} \end{bmatrix} \cdot \begin{bmatrix} \underline{u}_a \\ \underline{u}_b \end{bmatrix} = \begin{bmatrix} \underline{F}_a \\ \underline{F}_b \end{bmatrix} \tag{10}$$

If two elements with linear interpolation function are connected with the beam element the transformation matrix eq. (4) is .

$$\underline{T} = \begin{bmatrix} 1/4 & 0 & 1/2 & 0 & 1/4 & 0 \\ 0 & 1/8 & 0 & 3/4 & 0 & 1/8 \\ 1/d & 0 & 0 & 0 & -1/d & 0 \end{bmatrix} \qquad \underline{u}_T = \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix} \quad \text{and} \quad \underline{u}_S = \underline{T} \cdot \underline{u}_T . \tag{11}$$

Eq. (11) illustrates the averaging process of the transformation matrix for the displacements.

The transformation of node *a* by EST gives the modified stiffness matrix of the beam

$$\begin{bmatrix} \underline{T}^T \cdot \underline{K}_{aa} \cdot \underline{T} & \underline{T}^T \cdot \underline{K}_{ab} \\ \underline{K}_{ba} \cdot \underline{T} & \underline{K}_{bb} \end{bmatrix} \cdot \begin{bmatrix} \underline{u}_T \\ \underline{u}_b \end{bmatrix} = \begin{bmatrix} \underline{F}_T \\ \underline{F}_b \end{bmatrix} \tag{12}$$

Node *b* can be transformed similarly, if necessary.

The model neglects the local stiffness of the beam at the connection. It can be taken into account approximately by adding the stiffness of a beam with the height h/2 at the nodes connecting the beam and the shell elements, e.g. nodes 1-2 and 2-3 in Fig. 3. The model with the stiffness extension is called ESTS.

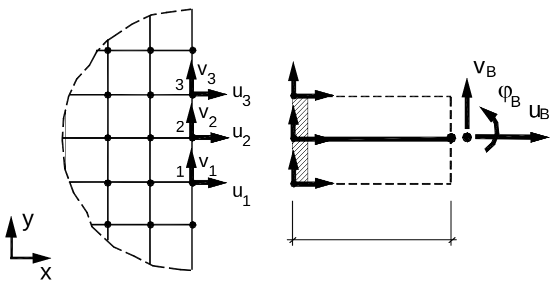

Figure 3: Connection of a beam with plane stress elements in an EST model (two element connection)

4. Application: Deep beam with columns

The Equivalent Stiffness Transformation can be applied to any beam-to-shell-connection. The case of a deep beam supported by two columns is considered, Fig. 4. The columns are modeled by

- finite elements (FEM)
- beams with a rigid multipoint constraint connection (MPC)
- beams with a EST connection
- beams with a ESTS connection

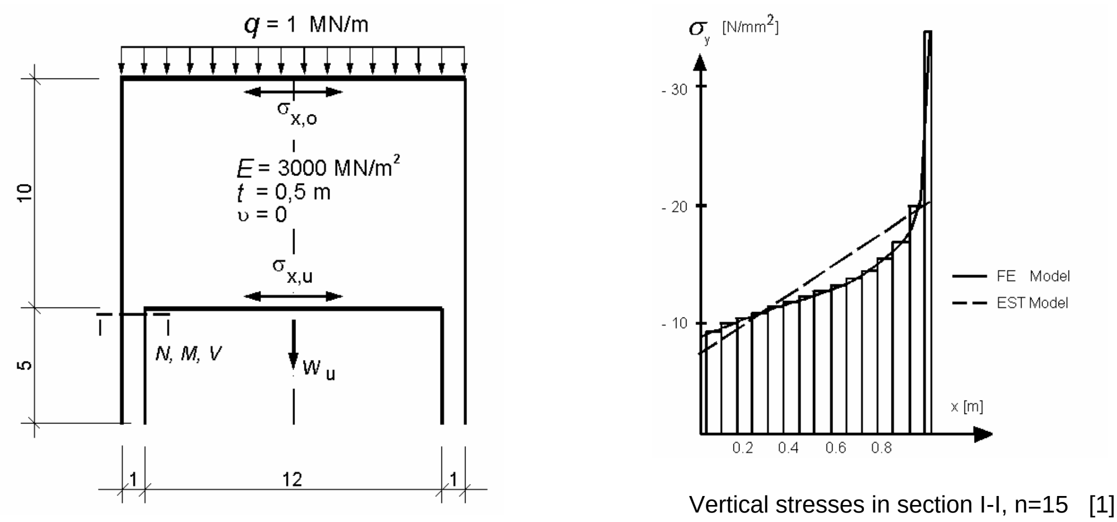

Figure 4: Deep beam

The element size is varied. Fig. 5 shows the finite element model for an element size of 0.5 [m] i.e. 2 elements per [m] or n=2. The stresses and the displacements of the FEM model at selected points agree well with the EST model, Tab. 1 [1].

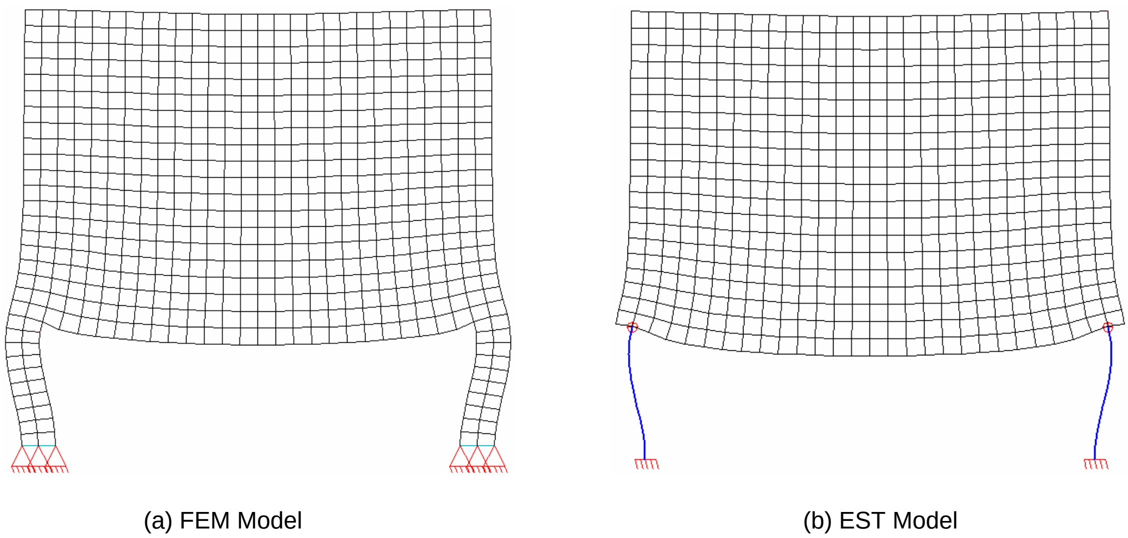

Figure 5: Deformed Finite Element Models (n=2)

Table 1: Stresses and displacements - FEM versus EST

stresses & displacements		n			
		1	2	4	8
FEM/ EST	$\sigma_{x,o}$ [N/mm²]	–1.80 / -1.83	–1.91 / -192	–1.94 / -195	–1.95/-1.96
	$\sigma_{x,u}$ [N/mm²]	3.51 / 3.59	3.60 / 3.63	3.64 / 3.65	3.65 / 3.66
	w_u [mm]	3.90 / 393	3.99 / 4.00	4.02 / 4.03	4.03 / 4.04

The convergence of the bending moment of the column in section I-I is shown in Fig. 6. It can be seen that the multipoint constraint model behaves too rigid whereas the EST model is too soft. The improved ESTS model converges to the finite element solution. For moderately fine meshing with n=2 to n=4, however, the EST model gives the best results. In this case the slight overestimation of the stiffness of the isoparametric plane stress elements is compensated by the slight underestimation of the stiffness by the pure EST model.

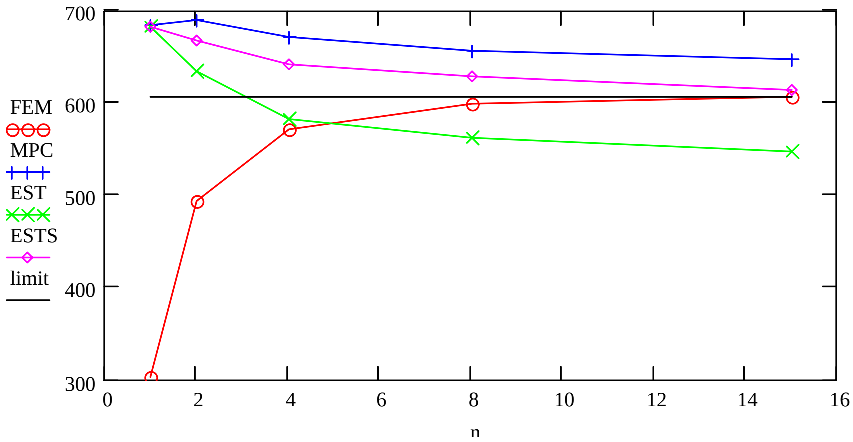

Figure 6: Convergence of the column bending moment [kNm] in section I-I

5. Conclusions

The EST is well suited to model the connection between domains of finite elements with different stress systems. The results agree well with more sophisticated finite element models. EST can be applied to connect structural elements with different stress systems as e.g. beams with plate elements. In EST-beams, the sectional forces which are required in RC design are obtained directly without numerical integration of element stresses. The model can be extended to any other type of beam to solid connection.

References

[1] M. Schneider, 2001. *Finite-Element-Berechnung von Scheibentragwerken*, Diploma Thesis, University of Applied Sciences Konstanz, Department of Civil Engineering (Prof. Dr. Werkle)

[2] H. Werkle, 2000. *Konsistente Modellierung von Stützen bei der Finite-Element-Berechnung von Flachdecken*, Bautechnik, pp 416-425, Berlin (Germany): Ernst&Sohn,

[3] H. Werkle, 2001. *Finite Elemente in der Baustatik*, 2nd Edition, Wiesbaden (Germany): Vieweg

[4] H. Werkle, 2002. *Analysis of flat slabs by the finite element method*, Advances in Structural Engineering and Mechanics (ASEM'02), Pusan (Korea)

[5] H. Werkle, 2002. *Modeling of connections between dissimilar finite element domains*, Fifth World Conference on Computational Mechanics (WCCM V), Vienna (Austria)