Effectiveness of "Detuned" TMD's for Beam-like Footbridges

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Abstract
The dynamic properties of Tuned Mass Dampers (TMD's) used to reduce the vibrations of footbridges are generally based on the optimization criterion of Den Hartog. However the spring, mass and damper of a TMD may possess slight fabrication tolerances or change its damping element properties due to temperature, which can result in a detuning of a TMD. The study presented deals with the influence of a detuning on the performance of a TMD. A simplified model for beam-like footbridges under pedestrian loading is proposed. It is shown that a 2-DOF-system in harmonic stationary motion represents a good approximation of the beam structure under transient time-dependent load. Graphs are given to assess the increase of acceleration and the decrease of effectiveness due to detuning. A case study on a footbridge with a span of 45 m demonstrates its accuracy.

Key words: Tuned Mass Damper, TMD, footbridges, pedestrian bridges, human-induces vibrations

1. Introduction

The mitigation of human-induced vibrations has become an important issue in the design of modern lightweight footbridges. The fundamentals of the excitation and the analytical methods for computation have been known since the 1980's. In the last decade many countries introduced serviceability requirements for human induced vibrations in their building codes. These define limit values for the admissible maximum accelerations. In cases where these limits cannot be kept for the bridge in design, Tuned Mass Dampers (TMD's) are appropriate means to reduce the vibrations considerably.

The dynamic properties of a TMD i.e. its mass, spring constant and damping are determined considering the modal mass and the eigenfrequencies of the bridge. They are generally based on an optimization criterion for harmonic stationary motion known as the Den Hartog criterion. However the parameters of the tuned mass damper may change during the lifetime of the structure which results in a detuning of the TMD. The paper deals with the influence of the detuning on the effectiveness of a TMD for beam-like bridges where the first eigenfrequency is excited by a pedestrian. First, harmonic stationary motions are investigated. The results are compared with a comprehensive analysis of a realistic footbridge in time domain, in particular taking the transient loading by pedestrians into account.

It should also be noted that the structural design parameters, e.g. the modal mass and the natural frequencies might differ from the values used in calculation and might change during the lifetime of the structure. These effects may also be a possible source of detuning which is not investigated here.

2. Analysis of Human-induced Vibrations of Footbridges

Humans may excite a bridge to vibrate by different actions, such as walking, running, jumping and vandalism. These actions are generally described by load models. Here it is presumed that there is no interaction between the action and the action effect, i.e. the movement of the bridge, also called "lock-in effect". A review of different models is given by Zivanovic (1) and Butz et. al. (2).

Walking of a single person excites forces on the ground in the vertical as well as in horizontal directions. In this paper only vertical excitations are considered. They may be modelled as a single load $F(t)$ varying in time and propagating with the velocity c as

$$c = f_S \cdot l_S \tag{1}$$

where f_s denotes the step frequency and l_S the step length. The step length depends on the step frequency and can be assumed to be 0.75-0.80m in the frequency range of about 2.0 Hz considered here. The time duration of a single step equals

$$T_S = \frac{1}{f_S} \tag{2}$$

The load time history is expressed by a Fourier series as

$$F(t) = G \cdot \left(1 + \sum_{j=1}^{4} \alpha_j \cdot \sin(2 \cdot \pi \cdot j \cdot f_S \cdot t - \varphi_j)\right) \tag{3}$$

where G denotes the weight of the person (e.g. 0.7 kN) and f_s the step frequency. The Fourier coefficients α_j, φ_j have been determined by different authors (see (1), (2)).

Here the coefficients given by Bachmann (3) have been used; see Table 1. Typical step frequencies for walking are between 1.7 and 2.3 Hz. In a dynamic analysis the step frequency has to be assumed to cause the most severe action effect, i.e. to be in resonance with the structure, if the structure possesses an eigenfrequency in that range.

Table 1. Fourier coefficients for walking

i	α_i	φ_i
1	$0.4 \ \ for \ \ f_s \le 2 \ \ Hz \quad 0.5 \ \ for \ \ f_s \ge 2.4 \ \ Hz$ $0.4 + 0.1 \cdot (f_S - 2)/0.4 \ \ for \ 2.0\,Hz \le f_s \le 2.4\,Hz$	0
2	0.1	π/2
3	0.1	π/2

The computational simulation of the vibrations of the bridge can be performed with the finite element method applying the described load model. For serviceability design the maximum acceleration computed for a single person pacing the bridge has to be augmented in order to consider a group of persons on the bridge, e.g. according to (4) for N persons walking on the bridge by the multiplication factor

$$m_{Group} = \sqrt{N} \tag{4}$$

Simplified formulae have been derived in order to easily assess the maximum vertical acceleration of beam-like bridges. For a 1-DOF system with mass $\overline{m}$, spring constant k and damping ratio ξ with a harmonic force $F(t) = F_0 \cdot \sin(\Omega \cdot t)$ caused by a single person, the stationary response for the maximum acceleration at resonance is

$$a_{vert,1} = \omega^2 \cdot u_{vert,1} = \frac{k}{\overline{m}} \cdot \frac{1}{2 \cdot \xi} \cdot \frac{F_0}{k} = \frac{F_0}{2 \cdot \overline{m} \cdot \xi} \tag{5}$$

For a simply supported beam with the mass per length m and the span width L the modal mass is $\overline{m} = m \cdot L / 2$. In order to take into account that the load is transient and moving on the beam with a velocity c instead of being a stationary harmonic load on a

1-DOF system, a reduction factor α_{red} is introduced. Hamm (5) suggests a factor $\alpha_{red}=0.75$. One obtains (6)

$$a_{vert,1}=\alpha_{red}\cdot\frac{F}{M\cdot\xi} \qquad (6)$$

with $\alpha_{red}=0.71$ and $F_0=0.4\cdot 700N=280N$ corresponding to the first Fourier term the formula of the maximum vertical acceleration for a single person given in Eurocode 5 (7) is obtained as

$$a_{vert,1}=\frac{200}{M\cdot\xi} \quad \text{for} \quad f_{vert}\le 2{,}5Hz \qquad (7)$$

where M denotes the total mass of the bridge in kg, ξ the damping ratio and f_{vert} the first eigenfrequency of the (one-span) bridge in vertical direction. It should be noted that the formula gives the maximum acceleration in resonance of a 1-DOF system with the modal mass of the first mode, a harmonic load F_0 and a reduction factor α_{red} without any further assumptions.

3. Tuned Mass Dampers

To reduce the vibrations of footbridges, Tuned Mass Dampers (TMD's) are generally used. They consist of a mass, a spring and a damper attached at the main system e.g. the footbridge. The main system with a TMD can be simplified as a 2-DOF system in which the bridge (or its modal mass) is represented as a 1-DOF system with the parameters k_H, c_H and m_H, as in Fig. 1.

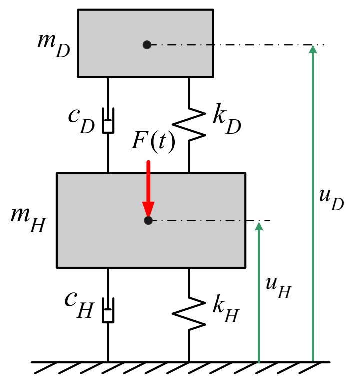


Fig. 1 Simplified model for a main system with a TMD.

The solution of its equation of motion for stationary harmonic motion is (2):

$$\bar{u}_H(t)=u_H\cdot sin(\Omega\cdot t+\varphi) \qquad (8)$$

and

$$\bar{\ddot{u}}_H(t)=-\Omega^2\cdot u_H\cdot sin(\Omega\cdot t+\varphi)=-\Omega^2\cdot\bar{u}_H(t) \qquad (9)$$

with the dynamic amplification ratio

$$V_H(\eta)=\frac{u_H}{u_{H,stat}}=\sqrt{\frac{A^2+B^2}{C^2+D^2}} \qquad (10)$$

and

$$A(\eta)=-\eta^2+\kappa^2,\quad B(\eta)=2\cdot\kappa\cdot\zeta_D\cdot\eta \qquad (11a)$$

$$C(\eta)=\eta^4-[1+(1+\mu)\cdot\kappa^2+4\cdot\kappa\cdot\zeta_H\cdot\zeta_D]\cdot\eta^2+\kappa^2 \qquad (11b)$$

$$D(\eta)=[-2\cdot[\zeta_H+(1+\mu)\cdot\kappa\cdot\zeta_D]\cdot\eta^3]+2\cdot\kappa\cdot(\kappa\cdot\zeta_H+\zeta_D)\cdot\eta \qquad (11c)$$

$$\eta=\frac{\Omega}{\omega_H},\quad \mu=\frac{m_D}{m_H},\quad \kappa=\sqrt{\frac{k_D\cdot m_H}{k_H\cdot m_D}},\quad \omega_H=2\cdot\pi\cdot f_H=\sqrt{\frac{k_H}{m_H}}$$

$$\xi_H=\frac{c_H}{2\cdot\sqrt{m_H\cdot k_H}},\quad \xi_D=\frac{c_D}{2\cdot\sqrt{m_D\cdot k_D}},\ u_{stat}=\frac{F_0}{k_H}. \qquad (11d)$$

The properties of the TMD are generally based on an optimization criterion for steady state motion as the Den Hartog criterion (8). The target of this optimization is the minimization of the displacements of the system with $\xi_H = 0$. According to Den Hartog the frequency ratio should be chosen to be

$$\kappa_{opt} = \frac{1}{1+\mu} \tag{12}$$

and the damping ratio as

$$\xi_{D,opt} = \sqrt{\frac{3\cdot\mu}{8\cdot(1+\mu)^3}} \tag{13}$$

The corresponding parameters of the damper are

$$m_D = \mu \cdot m_H \tag{14a}$$

$$f_{D,opt} = \kappa \cdot f_H \tag{14b}$$

$$k_{D,opt} = 4\cdot\pi^2\cdot m_D \cdot f_{D,opt}^2 \tag{14c}$$

$$c_{D,opt} = 4\cdot\pi\cdot m_D \cdot f_{D,opt}\cdot\xi_{D,opt} \tag{14d}$$

The main parameter for the design of the TMD is the mass ratio μ. In addition the mass and the eigenfrequency of the main structure have to be known. In practice, values of μ=0.02-0.08 are chosen. Fig. 2 shows the magnification function of the amplitudes of the main system for mass ratios μ=0.02, 0.05 and 0.08.

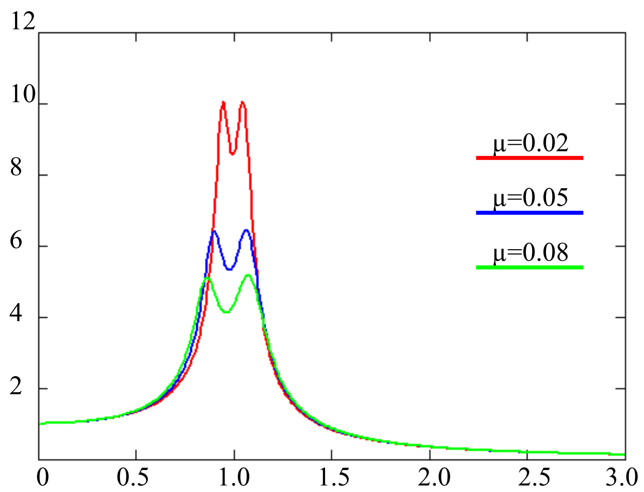


Fig. 2 Amplitudes for a main system with a TMD, μ=0.02, 0.05, 0.08 and $\xi_H = 0$.

With these parameters of the TMD the maximum dynamic amplification ratio for $\xi_H = 0$ is given by Den Hartog as

$$V_{H,\max} = \sqrt{1+\frac{2}{\mu}} \tag{15}$$

Using the maximum dynamic amplification factor of a 1-DOF-system which is $V_{H,max} = 1/(2\cdot\xi)$ an equivalent damping of the TMD can be determined as

$$\xi_{H,equival} = \frac{1}{2\cdot\sqrt{1+\frac{2}{\mu}}} \tag{16}$$

As this equation has been derived for $\xi_H = 0$ some portion of the damping of the main system e.g. $0.5\cdot\xi_H$ may be added.

The maximum acceleration of the main mass of the 2-DOF system acc. to eq. (9) is

$$a_{H,opt} = \max\left|\bar{\ddot{u}}_H(t)\right| = \Omega^2\cdot\left|\max\bar{u}_H(t)\right| = \omega_H^2\cdot\max\left|\eta^2\cdot V_H(\eta)\right|\cdot\frac{F_0}{k_H} \tag{17}$$

In the case of a beam-like bridge the equivalent damping may be used to determine the maximum acceleration system according to eq. (7). As comparative computations show due to the large damping of the system the reduction factor should be chosen to be $\alpha_{red} = 1$.

With this assumption one obtains for a system with a TMD designed according the Den Hartog criteria

$$a_{vert,1} = \frac{280}{M \cdot \xi_{H,equival}} = \frac{560}{M} \cdot \sqrt{1 + \frac{2}{\mu}} \quad \text{for} \quad f_{vert} \le 2{,}5Hz \tag{18}$$

4. Detuning of TMD's

In the production process springs, masses and dampers for the TMD are subjected to fabrication tolerances. In addition during lifetime there may be environmental influences. In particular, the outside temperature influences the damping constant of fluid dampers significantly, e.g. by a factor of 2 to 4, since the viscosity of the fluid is severely temperature dependent. Low temperatures (e.g. in winter) decrease the viscosity and increase the damping constant whereas high temperatures cause a loss of damping effect.

In order to investigate the influence of these parameters on the effectiveness of TMD's the mass, spring and damper are modified by the coefficients α_m, α_k and α_c:

$$\tilde{m}_D = \alpha_m \cdot m_D, \quad \tilde{k}_D = \alpha_k \cdot k_D, \quad \tilde{c}_D = \alpha_c \cdot c_D \tag{19}$$

and for the frequency ratio

$$\tilde{\kappa} = \sqrt{\frac{\alpha_k}{\alpha_m}} \cdot \kappa \tag{20}$$

The dynamic amplification ratio $\tilde{V}_H(\eta)$ of the detuned system can be determined with eq. (10) using the dimensionless parameters of the modified system

$$\tilde{\mu} = \alpha_m \cdot \mu, \quad \tilde{\kappa} = \sqrt{\frac{\alpha_k}{\alpha_m}} \cdot \kappa, \quad \tilde{\xi}_D = \frac{\alpha_c}{\sqrt{\alpha_m \cdot \alpha_k}} \cdot \xi_D \tag{21}$$

This gives the maximum acceleration of the detuned system

$$\tilde{a}_H = \omega_H^2 \cdot \max\left|\eta^2 \cdot \tilde{V}_H(\eta)\right| \cdot \frac{F_0}{k_H} \tag{22}$$

and the ratio of the accelerations of the detuned and the optimally tuned (according to Den Hartog) system

$$\beta_a = \frac{\max\left|\eta^2 \cdot \tilde{V}_H(\eta)\right|}{\max\left|\eta^2 \cdot V_H(\eta)\right|}. \tag{23}$$

The acceleration ratios for μ=0.02, 0.05, 0.08, $0.8 \le \alpha_m \le 1.2$, $0.8 \le \alpha_k \le 1.2$ and $\xi_H = 1\%$ are shown in Fig. 3.

For $\alpha_k / \alpha_m \approx 1$ the frequency ratio with $\tilde{\kappa} \approx \kappa$ is approximately unchanged and the maximum accelerations are near the accelerations for the tuning optimum or increase only slightly. There is even a small region where the accelerations slightly decrease compared to the "optimally" tuned TMD, e.g. at $\alpha_m = \alpha_k = 1{,}10$. This indicates that the Den Hartog optimization criteria which refers to the minimization of the displacements and not of the accelerations do not give the true minimum of the accelerations. For detuned systems with $\alpha_k / \alpha_m << 1$ or $\alpha_k / \alpha_m >> 1$ the accelerations increase significantly, especially for low mass ratios μ.

In order to assess the efficiency of the detuned damper the ratio

$$\nu = \frac{a_0 - \tilde{a}_H}{a_0 - a_{H,opt}} \tag{24}$$

is introduced, where a_0 is the maximum acceleration of the system without TMD. If the detuned system has the same maximum acceleration as the optimally tuned system, i.e. if $\tilde{a}_H = a_{H,opt}$ the effectiveness ratio is 1. If the maximum acceleration of the detuned system is the same as for the system without TMD, i.e. $\tilde{a}_H = a_0$, the efficiency is 0.

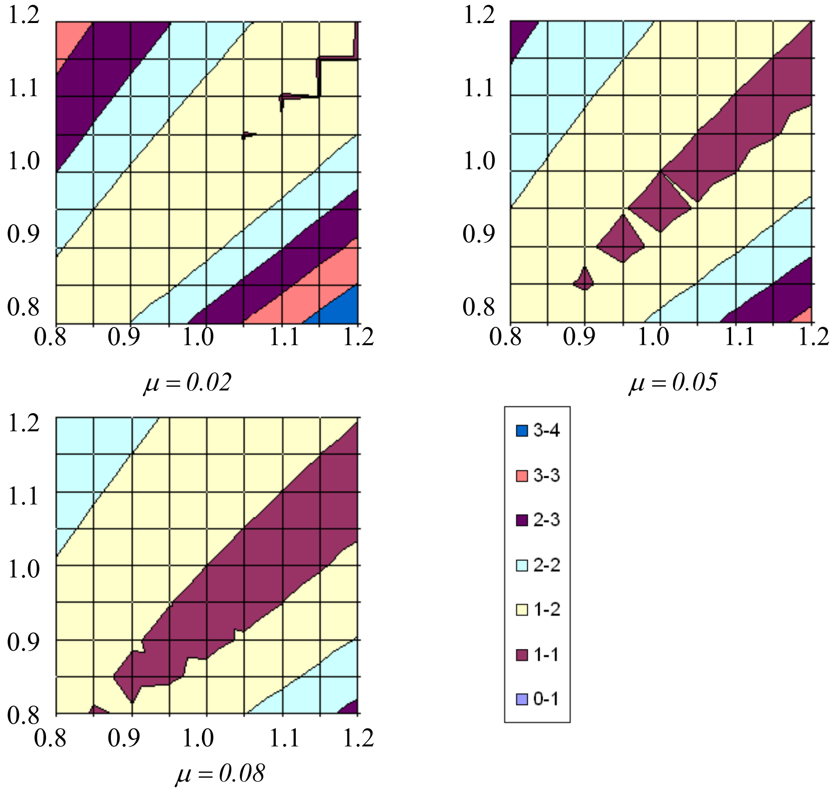


Fig. 3 Acceleration ratios β_a of detuned TMD's – detuning of the spring and the mass.

First the case of the 2-DOF-system is investigated. For the maximum acceleration of the system without TMD we obtain according to eq. (5) and the reduction factor $\alpha_{red} = 0.75$:

$$a_0 = \alpha_{red} \cdot \omega_H^2 \cdot u_{vert,1} = \omega_H^2 \cdot \frac{\alpha_{red}}{2 \cdot \xi_H} \cdot \frac{F_0}{k_H} \qquad (25)$$

Introducing eq. (25) and eqn's (17), (22) in (24) the effectiveness ratio of a detuned 2-DOF-system for harmonic stationary motion is obtained as

$$\nu = \frac{\dfrac{\alpha_{red}}{2 \cdot \xi_H} - max\left|\eta^2 \cdot \tilde{V}_H(\eta)\right|}{\dfrac{\alpha_{red}}{2 \cdot \xi_H} - max\left|\eta^2 \cdot V_{H,opt}(\eta)\right|} \qquad (26)$$

Fig. 4 shows the effectiveness of detuned TMD's for μ=0.02, 0.05, 0.08, variation of the mass and spring parameters of $0.8 \le \alpha_m \le 1.2$ and $0.8 \le \alpha_k \le 1.2$ and damping ratio of the main system $\xi_H = 1\%$. The figure shows that the effectiveness of the TMD is hardly influenced if $\kappa_\kappa = \sqrt{\alpha_k / \alpha_m} \approx 1$ but it is sensitive to ratios $\alpha_k / \alpha_m << 1$ or $\alpha_k / \alpha_m >> 1$ for low mass ratios μ. Again it can be noted that there is a small range where the Den Hartog criterion does not represent the real optimum solution for the accelerations, giving effectiveness ratios $\nu > 1$.

Figures 3 and 4 can be used to assess the influence of detuning of the spring and the mass of a TMD of a footbridge on the accelerations.

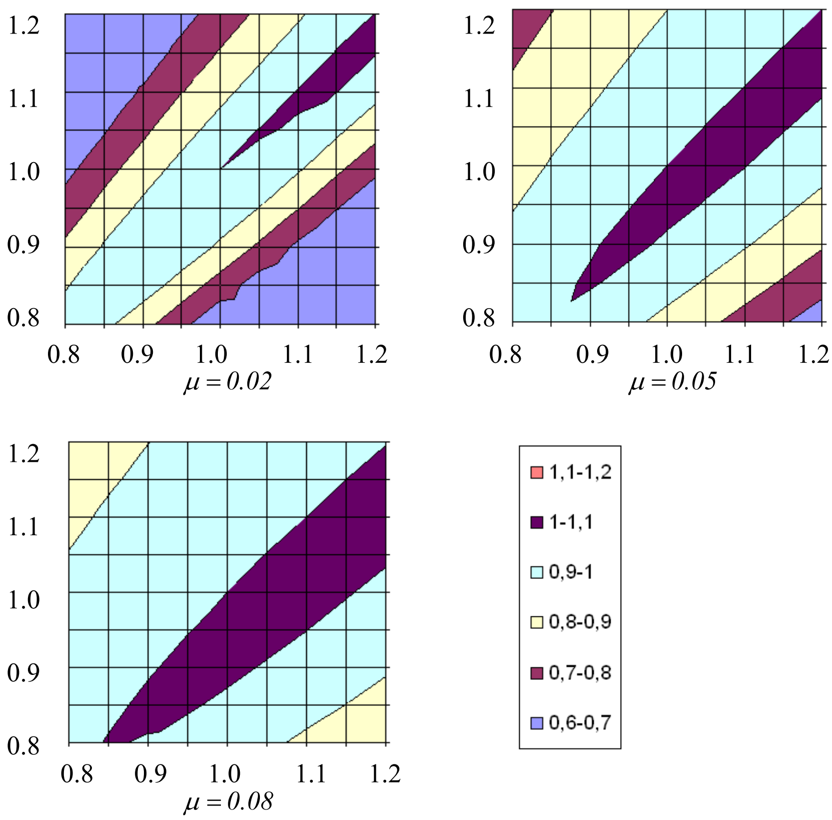


Fig. 4 Effectiveness of detuned TMD's – detuning of the spring and the mass.

Damping coefficients possess a much larger range of variability than springs and masses. Fig. 5 shows the influence of the damper coefficient on the effectiveness of a TMD. Even for a large increase of the damping coefficient the effectiveness decreases only moderately whereas for a large decrease the effectiveness decreases fatally. This effect should be taken into account in a robust design of dampers for TMD's.

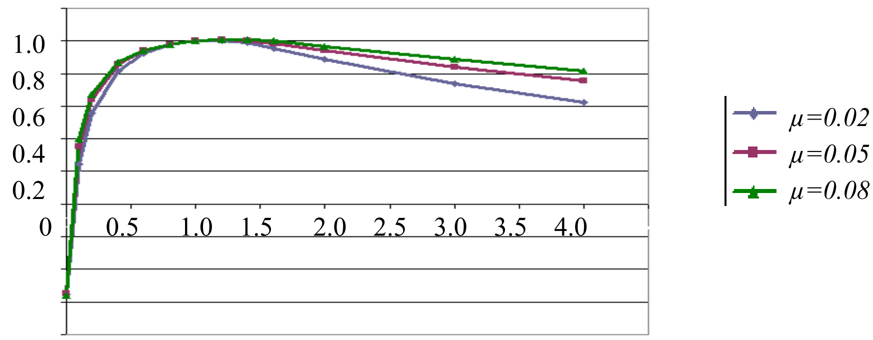


Fig. 5 Effectiveness of detuned TMD's – detuning of the damper.

5. Case Study

The influence of a possible detuning is studied on a footbridge in Basel, Switzerland; see Fig. 6. It has a span width of 45 m, a total mass of 160 t and a first eigenfrequency of 1.75 Hz. The damping of the steel bridge is 1%.

Fig. 6 Stückisteg footbridge, Basel, Switzerland.

A finite element analysis has been performed on a simplified beam model with constant cross section having the same first eigenfrequency as the original bridge. The bridge is loaded by a time-dependent point load as given in eq. (2) propagating with the velocity c acc. to eq. (1) on the bridge (9). The weight of the person is assumed to be 0.7kN, the step length 0.80m and the step frequency equals the first eigenfrequency of 1.75Hz of the bridge. Fig. 7 shows the time history of the vertical accelerations in the middle of the bridge.

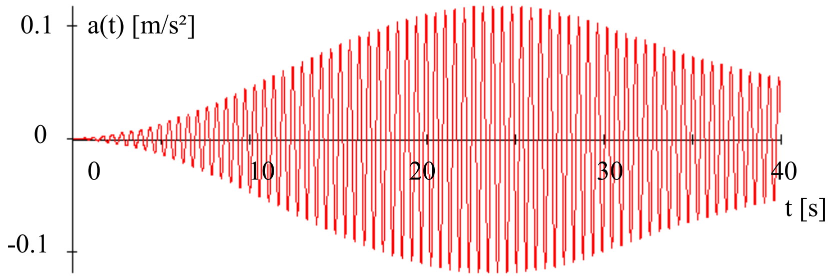


Fig. 7 Acceleration time history at the middle of the bridge without TMD, a_max=0.14m/s².

Now a TMD is added to the finite element model in the middle of the bridge. Its parameters are determined according to the Den Hartog optimization criteria. The lowest eigenfrequencies are now obtained as 1.61 Hz and 1.86 Hz. With a TMD with $\mu=0.02$ (referring to a modal mass of 80 t) and a step frequency of 1.86 Hz the vertical accelerations in Fig. 8 are obtained.

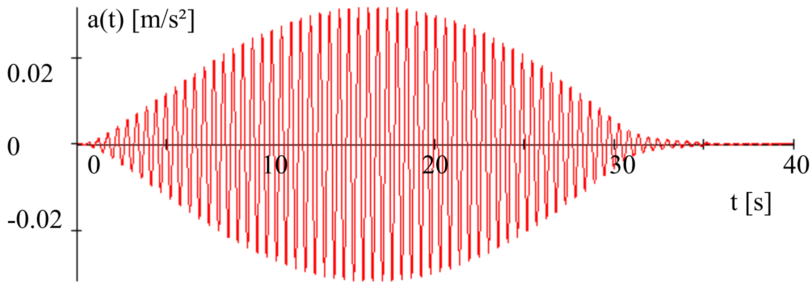


Fig. 8 Acceleration time history at the middle of the bridge with TMD, μ=0.02, a_max=0.032m/s².

The maximal vertical accelerations show a good agreement with the approximate relationships eq. (7) and eq (18), respectively; see Table 2.

Table 2. Maximum vertical acceleration in m/s² of the bridge

Analysis	without TMD	with TMD		
		μ=0.02	μ=0.05	μ=0.08
FEM	0.14	0.032	0.023	0.019
Eq. (7) and (18) resp.	0.13	0.035	0.022	0.018

The influence of a possible detuning of the TMD is studied by varying the mass coefficient α_m and the spring coefficient α_k separately. The acceleration coefficients β_a and effectiveness ratio ν, given in Fig. 9 for $\mu=0.02$, show a good agreement between the 2-DOF system in harmonic stationary motion and the FEM model under transient loading. The analysis with the FEM model again demonstrates that the Den Hartog criterion does not give the minimum accelerations which can be achieved with a TMD.

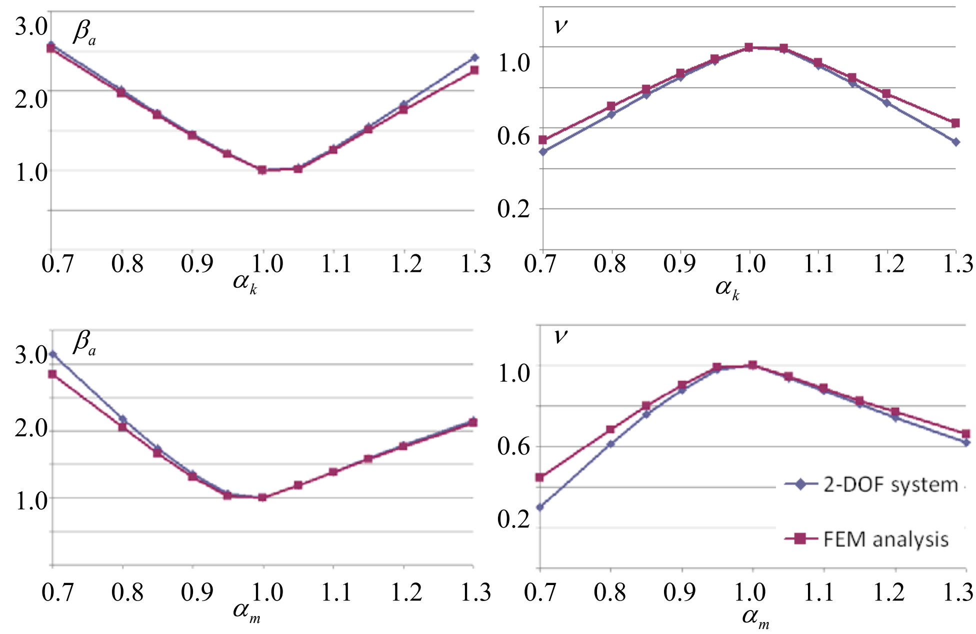
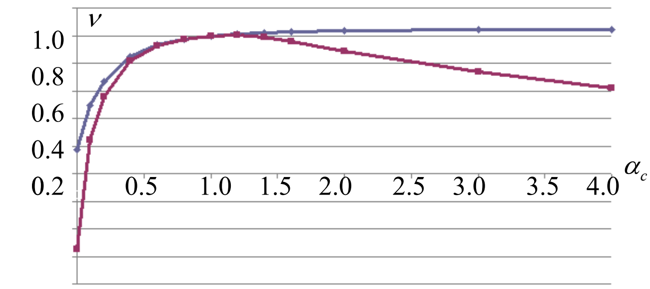
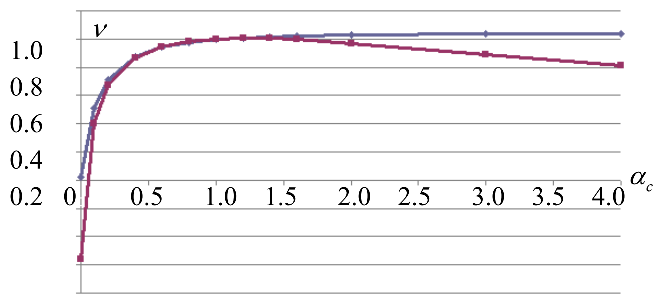


Fig. 9 Acceleration coefficient β_a and effectiveness ratio ν – detuning of the spring, μ=0.02.

The detuning of the damper is studied at the FEM model by varying the damper coefficient α_c in the range of 0 to 4. Results are shown in Fig. 10. They agree well for both models in the range of $0.2 < \alpha_c < 1.5$. However it is interesting to note that the results of the 2-DOF system for harmonic stationary motion underestimate the effectiveness for large coefficients α_c.



μ=0.02.



$\mu=0.08$

Fig. 10 Effectiveness ratio ν for a detuning of the damper.

6. Conclusions

The detuning of a TMD significantly influences its performance. However, even for considerably large deviations of the mass, stiffness and damper values an effectiveness of 80% to 90% is obtained. To assess the effects due to detuning, a 2-DOF-system in harmonic stationary motion can be used as a close approximation for beam-like footbridges. From the graphs in Figs. 3, 4 and 5, the increase of the accelerations and the decrease of the effectiveness, respectively, can readily be observed. The study deals with beam-like footbridges. It should be extended to bridges with a more sophisticated geometry and to other optimization criteria.

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