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MODELING OF DYNAMIC SOIL-STRUCTURE-INTERACTION IN THE THREE-DIMENSIONAL FINITE ELEMENT ANALYSIS OF BUILDINGS

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ABSTRACT

In the earthquake analysis of three-dimensional finite element models of buildings consisting of shell elements, soil-structure interaction may significantly influence the structural response. The paper compares how different models include soil-structure-interaction into the structural model taking into account the flexibility of the foundation slab as well as the stiffness and radiation damping of the soil. For dynamic analyses soil is modelled using the scaled boundary finite element method (SBFEM) in time domain. In addition, a simplified model based on analytical or semi-analytical solutions in frequency domain is investigated. Distributed springs and dampers (DSD) are used. The global response of the three-dimensional model of the building is compared with the response of a corresponding beam model. Both, time history analyses and response spectra analyses have been performed.

The study shows that soil-structure interaction may have a significant impact on the global response as well as on local sectional forces of three-dimensional finite element models of buildings. The results of the simplified model agree considerably well with the more sophisticated scaled boundary finite element method. However, the DSD-model is easier to handle and can take into account any given layering of the soil, whereas the scaled boundary finite element method is available for an elastic half-space only.

INTRODUCTION

For earthquake analyses, buildings are often represented by beam models. In beam models soil-structure interaction (SSI) may be taken into account by frequency dependent spring and damper elements representing the dynamic impedance functions of a rigid foundation on soil. The earthquake response of the structure is obtained by a time history analysis in frequency domain. In order to perform the analysis more conveniently, in time domain simplified assumptions for the soil behaviour as modal damping must be made. The concept of modal damping also allows a response spectrum analysis of the structure (Werkle, 2008).

In structural analysis of buildings for static actions, three-dimensional finite element models of the complete building consisting mainly of shell elements are becoming more and more popular. In the earthquake analysis of those three-dimensional finite element models, the influence of SSI often is neglected, i.e. a rigid base is assumed at the foundation level. In the case of large stiff structures on soft soils, however, SSI significantly influences the structural response and must not be omitted (Veletsos and Meek, 1975; Wolf, 1994). The modelling of SSI for three-dimensional shell models,

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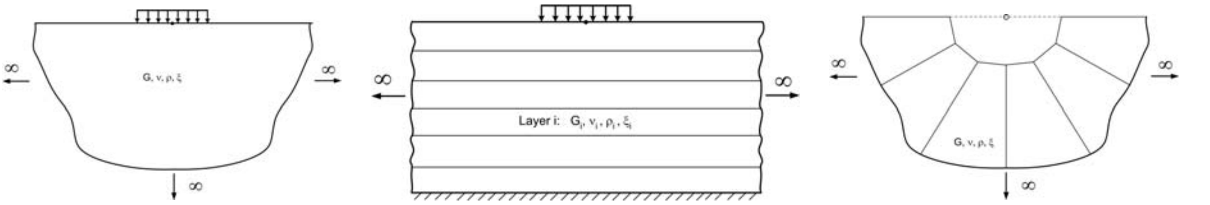
however, is more complicated than in the case of beam models. It should take into account the flexibility of the foundation slab as well as the stiffness and the radiation damping in soil. Standard software solutions for modelling these important effects are rarely available. The paper discusses possible solutions and compares the results for a typical building.

SOIL MODELS

The analysis of SSI is generally based on Green's functions describing the dynamic displacements of the soil caused by a point force acting at the soil surface. Green's functions have been given for a half-space as well as for layered soils mostly in frequency domain (e.g. see Kausel 1981, Waas et al. 1985). They may be used to construct a stiffness matrix of the soil related to the nodal points connecting the soil with the foundation slab of the building. This procedure for the analysis of layered soils in frequency domain has been implemented in SASSI 2000 (Lysmer et al. 2006). As large finite element models of the building including soil stiffness are computed in frequency domain, the method is cumbersome and requires specialized software, however it is also very powerful. Green's functions in frequency domain can be used to compute the dynamic impedance functions of a rigid foundation, i.e. its frequency dependent spring and damper constants. Solutions are available for circular and rectangular rigid foundations on a homogeneous half-space and on layered soils (see e.g. Gazetas G., 1983; Sieffert and Cevaer, 1995; Werkle, 1988). The Thin-Layer-Method acc. to Kausel (1981) and Waas et al. (1985) as well as the "direct stiffness method" by integration over wavenumbers (see Schevels et al. 2010) are well suited to determine frequency dependent impedance functions for arbitrarily layered soils (Cibotaru M.A. and Werkle H., 2012).

Another approach for including SSI in dynamic finite element analyses is by using numerical methods. Soil can be represented by finite elements in frequency domain with transmitting, i.e. non-reflecting, boundaries for a layered soil allowing for wave propagation (FEM), see e.g. Werkle, 1986 and 1987. Other options are the Boundary Element Method (BEM) and the Scaled Boundary Finite Element Method (SBFEM) for an elastic half-space in frequency or in time domain, see Wolf (2003). Some models are shown in Figure 1. In this study the SBFEM in time domain as described by Radmanovic and Katz (2010) and implemented in the finite element Software ASE/DYNA by SOFiSTiK (2010) is used. The results obtained by this method are compared with the distributed spring and damper method (DSD).



(a) Homogeneous half-space (b) Layered Soil (c) SBFEM for homogeneous half-space

Figure 1. Soil Models

DSD METHOD

Buildings with a foundation slab typically stiffened by walls in the basement floor are assumed to have a stiffness similar to a rigid foundation slab, i.e. deviations of displacements of the foundation from rigid body motions of the foundation are neglected in the global soil stiffness and damping. Hence the impedance functions for a rigid foundation describe the global stiffness and damping of the foundation. They can be written in frequency domain with the circular frequency of vibration $\omega = 2 \cdot \pi \cdot f$ as

$$\widetilde{K}(\omega) = K_1(\omega) + i \cdot K_2(\omega) = K_1(\omega) + i \cdot \omega \cdot C_1(\omega) \tag{1}$$

For a rigid circular foundation with radius r on an elastic half-space the impedance function is given by

horizontal $$\widetilde{K}_h(\omega) = K_{1,h}(\omega) + i \cdot K_{2,h}(\omega) = K_{stat,h} \cdot \left(k_h(\omega) + i \cdot a_0 \cdot c_h(\omega)\right) \tag{1a}$$

rocking $$\widetilde{K}_r(\omega) = K_{1,r}(\omega) + i \cdot K_{2,r}(\omega) = K_{stat,r} \cdot \left(k_r(\omega) + i \cdot a_0 \cdot c_r(\omega)\right) \tag{1b}$$

with $a_0 = \omega \cdot r / v_S$. Here $v_S = \sqrt{G/\rho}$ (G= shear modulus, ρ = density) is the shear wave velocity in the half-space. The static spring constants are

horizontal $$K_{stat,h} = \frac{8 \cdot G \cdot r}{2 - \nu} \tag{2a}$$

rocking $$K_{stat,r} = \frac{8 \cdot G \cdot r^3}{3 \cdot (1 - \nu)} \tag{2b}$$

with the Poisson ratio ν of the soil. The frequency dependent coefficients $k_h(\omega)$, $k_r(\omega)$ and $c_h(\omega)$, $c_r(\omega)$ are given in Figure 2 (Gazetas 1983).

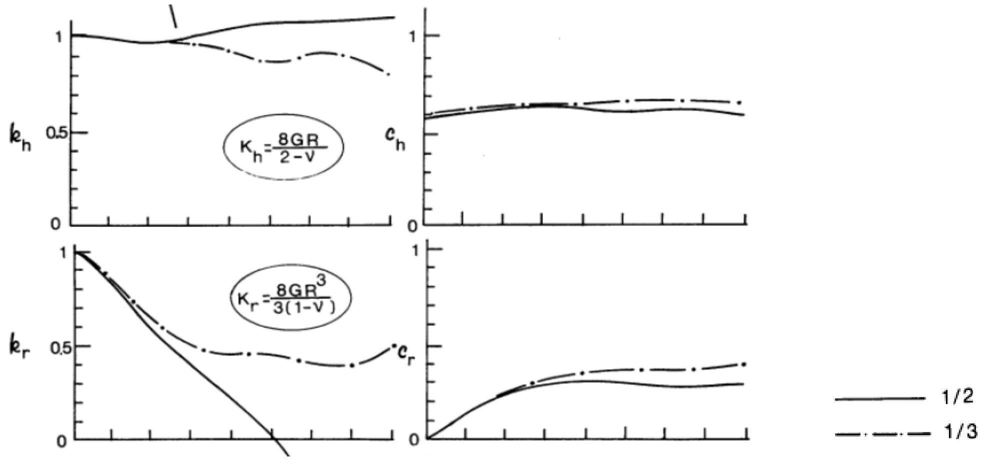


Figure 2. Impedance functions of an elastic half-space for $\nu = 1/3$ and $\nu = 1/2$

The equations can be understood as frequency dependent springs.

horizontal $$K_h(\omega) = K_{h,1}(\omega) = K_{stat,h} \cdot k_h(\omega)\,, \tag{3a}$$

rocking $$K_r(\omega) = K_{r,1}(\omega) = K_{stat,r} \cdot k_r(\omega) \tag{3b}$$

and dampers

horizontal $$C_h(\omega) = C_{h,1}(\omega) = \frac{K_{h,2}(\omega)}{\omega} = K_{stat,h} \cdot c_h(\omega) \cdot \frac{a_0}{\omega}\,, \tag{4a}$$

rocking $$C_r(\omega) = C_{r,1}(\omega) = \frac{K_{r,2}(\omega)}{\omega} = K_{stat,r} \cdot c_r(\omega) \cdot \frac{a_0}{\omega}\,. \tag{4b}$$

The springs and dampers of non-circular foundations can be approximated by a circular foundation having the same area A_F or the second moment of area I_F for horizontal and rocking motion, respectively, as an equivalent circular foundation. A_F and I_F relate to the contact area of the foundation and soil. Hence the equivalent radii $r_h = \sqrt{A_F / \pi}$ $\quad r_r = \sqrt[4]{4 \cdot I_F / \pi}$ are obtained for horizontal and rocking motion, respectively.

For the three-dimensional model of the building, these global values are be transformed to the nodal points of the finite element model of the foundation slab. Assuming a linear stress distribution of the soil stresses acting on the foundation, the transformation can be done by defining a distributed spring or modulus of subgrade reaction

horizontal $$K_{SG,h}(\omega) = \frac{K_{h,1}(\omega)}{A_F} \tag{5a}$$

rocking $$K_{SG,r}(\omega) = \frac{K_{r,1}(\omega)}{I_F} \tag{5b}$$

The corresponding distributed damping is obtained as

horizontal $$C_{SG,h}(\omega) = \frac{C_{h,1}(\omega)}{A_F} = \frac{K_{h,2}(\omega)}{\omega \cdot A_F} \tag{6a}$$

rocking $$C_{SG,r}(\omega) = \frac{K_{r,1}(\omega)}{I_F} = \frac{K_{r,2}(\omega)}{\omega \cdot I_F} \tag{6b}$$

These distributed stiffness and damping values are transformed to nodal point springs and dampers by standard finite element techniques (Figure 3b).

In cases where vertical motion, rocking about two axes as well as horizontal motion in two directions and torsional motion are considered simultaneously, the soil reaction cannot be described by a unique subgrade modulus. Instead, the equivalent stress transformation as given by Werkle (2002) may be applied, leading to a fully coupled stiffness matrix for the nodal points of the foundation slab instead of a diagonal matrix obtained for the distributed springs and dampers acc. to eqn's (2) to (5).

METHODS OF ANALYSIS

In this study time history analyses as well as response spectra analyses are performed. The modelling of SSI depends on the method of analysis (Table 1). Frequency dependant impedance functions are suited for computations in frequency domain only. For time history analyses the modelling of soil by the SBFEM in time domain is straightforward (Figure 3a).

Table 1. Modelling of SSI

Method of Analysis	**Soil-Structure Interaction**		
	Frequency dependent impedance functions	Constant impedance functions (mod. damp)	Time dependent reaction forces (SBFEM)
Time history analysis in frequency domain	x	(x)	-
Time history analysis in time domain	-	x	x
Response spectrum analysis	-	x	-

In the SBFEM the forces $p_I(t)$ at the interface between the unbounded region of the soil and the irregular soil region which is discretized in finite solid elements are given in time domain by

$$r_I(t) = \int_0^t M_I^\infty \cdot \ddot{u}_I(t-\tau)\, d\tau \tag{7}$$

where M_I^∞ is the acceleration unit-impulse matrix (Radmanovic, 2009). The equations of motion of the finite element model are (Figure 3a)

$$\begin{bmatrix} M_{SS} & M_{SI} \\ M_{IS} & M_{II} \end{bmatrix} \cdot \begin{bmatrix} \ddot{u}_S(t) \\ \ddot{u}_I(t) \end{bmatrix} + \begin{bmatrix} C_{SS} & C_{SI} \\ C_{IS} & C_{II} \end{bmatrix} \cdot \begin{bmatrix} \dot{u}_S(t) \\ \dot{u}_I(t) \end{bmatrix} + \begin{bmatrix} K_{SS} & K_{SI} \\ K_{IS} & K_{II} \end{bmatrix} \cdot \begin{bmatrix} u_S(t) \\ u_I(t) \end{bmatrix} = \begin{bmatrix} p_S(t) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ r_I(t) \end{bmatrix} \tag{8}$$

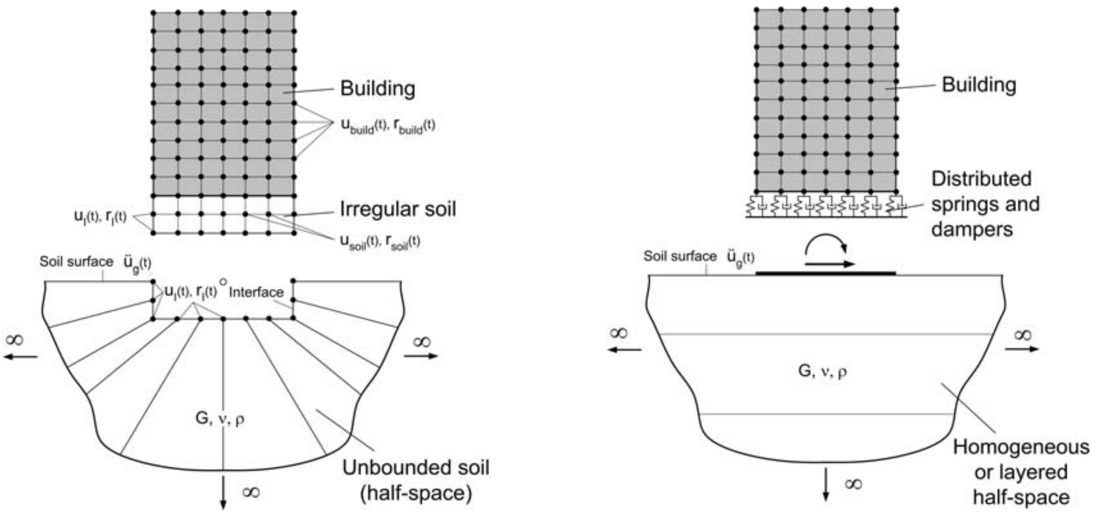
The mass, damping and the stiffness matrices are subdivided in the degrees of freedom relating to the structure and to the interface. The structure consists of the irregular region of the soil and of the building both discretized in finite elements, hence

$$u_S(t) = \begin{bmatrix} u_{Build}(t) \\ u_{Soil}(t) \end{bmatrix}, \quad p_S(t) = \begin{bmatrix} p_{Build}(t) \\ 0 \end{bmatrix}. \tag{9}$$

The earthquake acceleration time history $\ddot{u}_g(t)$ is defined at the soil surface. Therefore

$$p_{Build}(t) = -M_{Build} \cdot I_x \cdot \ddot{u}_g(t) \tag{10}$$

where M_{Build} is the mass matrix of the building and I_x the influence vector (elements are "1" in the direction of the earthquake acceleration and "0" in all other degrees of freedom).
It should be noted that the SBFEM in time domain is developed for an elastic halfspace without internal material damping, i.e. only radiation damping is considered in the analysis.



(a) (b)

Figure 3. Models for soil-stucture interaction; (a) SBFEM model (Radmanovic and Katz 2010); (b) DSD model

The SBFEM has been developed for an elastic half-space. For layered soils, however, SBFEM solutions are not available. Here, impedance functions in frequency domain acc. to eqn.'s (3) and (4) are approximately assumed to be constant, i.e. frequency independent. They are evaluated at the first eigenfrequency for horizontal and rocking motion of the building, respectively, and added to the damping matrix of the building.

In modal analysis, damping coefficients can be defined for each mode of vibration. In order to take into account the different damping behaviour of soil and building approximately equivalent modal damping coefficients can be defined (see e.g. Tsai, 1974). The modal damping coefficient ξ_{eq} in the *i*-th mode is obtained as weighted average of the damping coefficients of the building ($\xi_{B,i}$) and of the soil springs and dampers.

$$\xi_i = \frac{E_{h,i} \cdot \xi_{h,i} + E_{r,i} \cdot \xi_{r,i} + E_{B,i} \cdot \xi_{B,i}}{E_{h,i} + E_{r,i} + E_{B,i}} \tag{11}$$

The weighting factors are the potential energy in the building ($E_{B,i}$) and in soil springs for horizontal ($E_{h,i}$) and rocking ($E_{r,i}$) motion, respectively, related to the total potential energy in the i-th mode. The damping coefficients for the horizontal and rocking motion of a rigid foundation at the eigenfrequency ω_i are given by Werkle (2008) as

Horizontal $$\xi_{h,i} = \frac{a_{0,h}}{2} \cdot \frac{c_h(\omega_i)}{k_h(\omega_i)} + \xi_{soil} \,, \tag{12a}$$

Rocking $$\xi_{r,i} = \frac{a_{0,r}}{2} \cdot \frac{c_r(\omega_i)}{k_r(\omega_i)} + \xi_{soil} \,. \tag{12b}$$

with $a_{0,h} = \omega_i \cdot r_h / v_S$ and $a_{0,r} = \omega_i \cdot r_r / v_S$. The damping coefficient ξ_{soil} denotes the material damping of the soil.

In a response spectrum analysis acc. to Eurocode 8 modal damping may be taken into account by modifying the spectrum with $\eta = \sqrt{10/(5+\xi_{eq})} \geq 0.55$. This means that damping is limited to 28% due to approximate character of the method.

STRUCTURAL MODEL

A three story building over a basement floor with a floor plan similar to the building analysed by Fajfar and Kreslin (2011) has been investigated (Figure 4). The foundation slab has a size of 18 x 21 m, the total height of the building is 13 m. A floor plan of the first floor is shown in Figure 5. The walls have a thickness of 0,30 m, columns have a size of 0,3 x 0,3 m, the slab thickness is 0,20 m and the foundation slab thickness is 0,40 m (in the case of the ‘rigid’ foundation slab a thickness of 6,00 m is assumed without augmenting the foundation slab mass). The building consists of concrete with a Young’s Modulus of E = 28300 MN/m² and a material damping coefficient of 5%. The building is symmetric to the y-axis and unsymmetrical to the x-axis. In this study, only earthquake action in y-direction is considered.

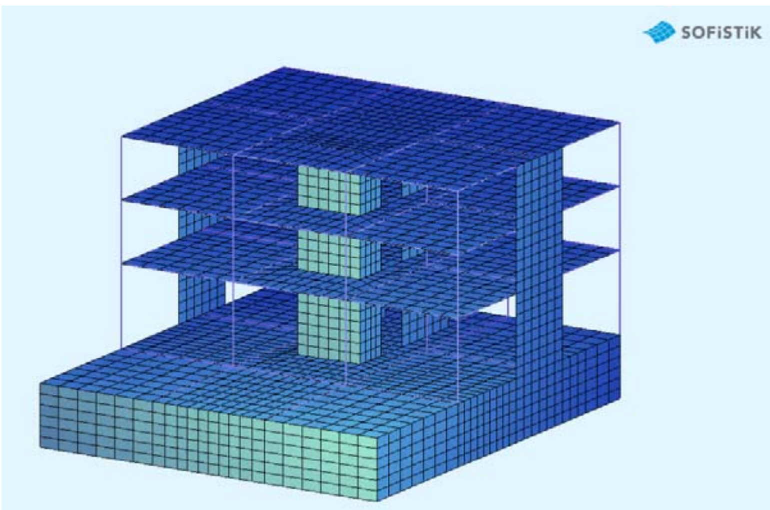


Figure 4. Three-dimensional finite element model of a multi-storey building (Volarevic, 2013)

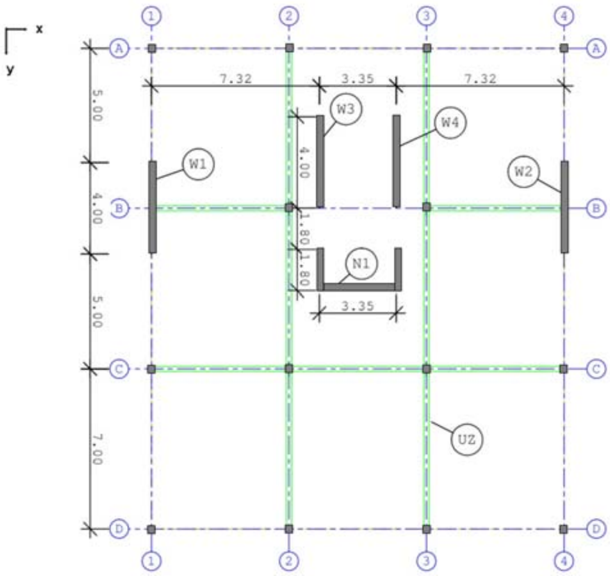
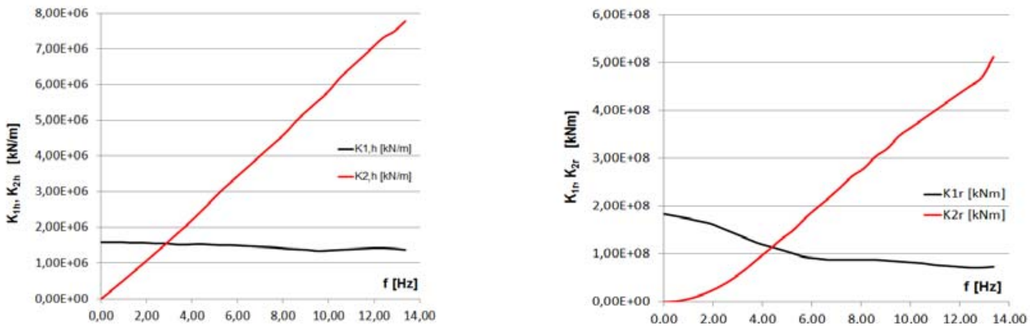


Figure 5. Floor plan

The soil consists of soft clay with a shear modulus of 30 MN/m² and a Poisson ratio of 0,33 i.e. a shear wave velocity of 129 m/s. Its density is 1,8 to/m^3. The material damping ξ_{soil} in soil cannot be modelled by the SBFEM in time domain. Hence, enabling comparison with other models, it is neglected in all methods.

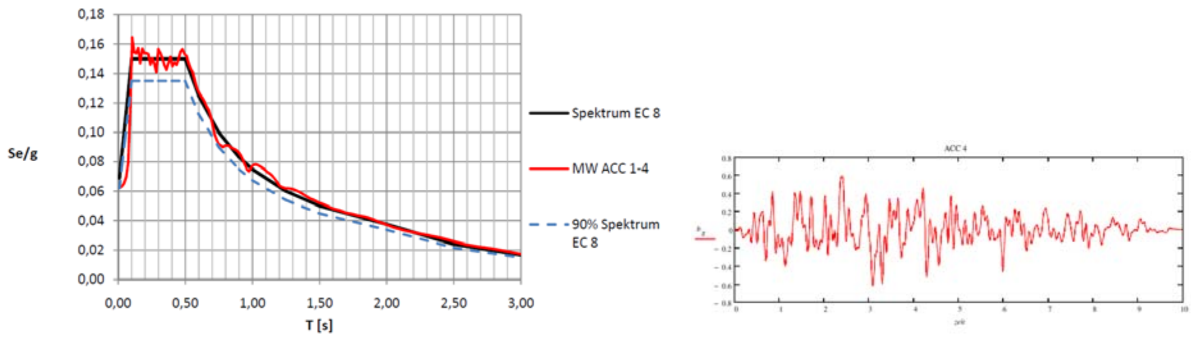
The static soil spring constants as given in eqn. (2a,b) are $K_{stat,h} = 1{,}58 \cdot 10^6\ kN/m$ and $K_{stat,r} = 1{,}83 \cdot 10^8\ kNm$, the frequency dependent impedance functions $K_{1,h}(\omega)$, $K_{2,h}(\omega)$ and $K_{1,r}(\omega)$, $K_{2,r}(\omega)$ acc. to eqns (1a,b) are shown in Figure 6.



(a) Horizontal (b) Rocking

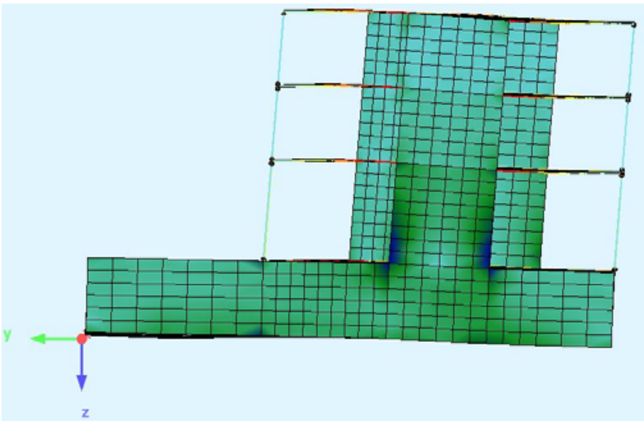
Figure 6. Impedance functions of the foundation

Earthquake action is defined at the interface between the soil and the structure, i.e. at the soil surface. An elastic response spectrum acc. to Eurocode 8 (DIN EN 1998-1/NA, 2011) for a ground type C-S, a ground peak acceleration $a_{gR} = 0.8\ m/s^2$ and importance factor $\gamma_I = 1.0$ is assumed. A spectrum compatible time history determined according to Meskouris et. al (2011) is given in Figure 7. For each time history analysis the mean value of 4 spectrum compatible time histories are evaluated.

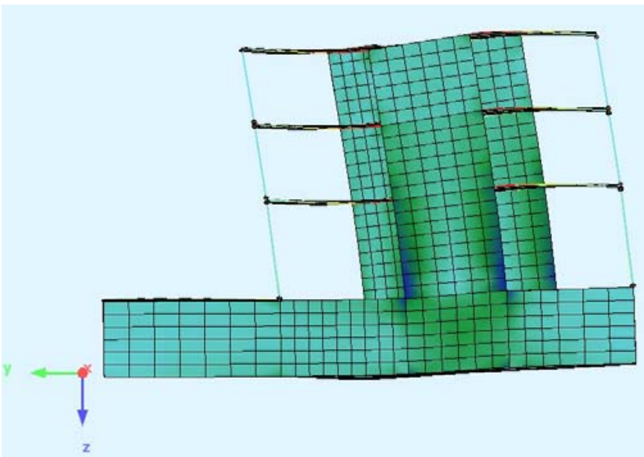


(a) (b)

Figure 7. Earthquake action (a) Acceleration response spectrum (b) Time history



(a) First mode in y-direction, f = 3.18 Hz



(b) Second mode in y-direction, f = 8.33 Hz

Figure 8. Mode shapes of the DSD model with flexible foundation slab

GLOBAL RESPONSE

To investigate the influence of soil-structure interaction and of the flexibility of the foundation slab on the global response of the structure different types of soil-structure interaction, various models have been studied. The following models have been considered:

- Model with a fixed base (without SSI)
- Model with rigid foundation slab (DSD and SBFEM models)
- Model with flexible foundation slab (DSD and SBFEM models)

For vibrations in y-directions the eigenfrequencies given in Table 2 are obtained. It can be noted that due to the soft soil conditions soil-structure interaction significantly influences the eigenfrequencies. The first two modes are considered since their modal masses represent approximately the total mass of the building. The mode shapes of the two modes in y-direction for the model with the flexible foundation slab are given in Figure 8.

Table 2. Eigenfrequencies and modal damping coefficients

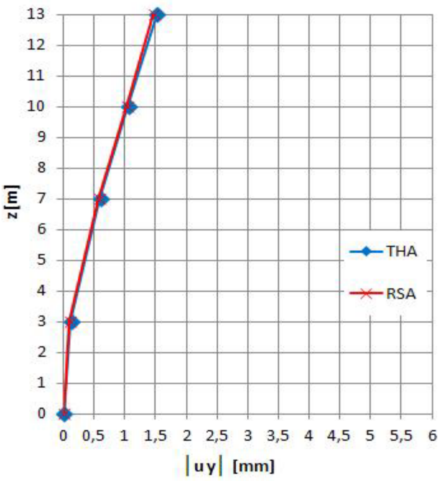
Soil-structure interaction model	**Eigenfrequencies [Hz]**		**Modal damping coefficients**	
	1. mode	2. mode	1. mode	2. mode
Fixed base (without SSI)	5,84	23,40	-	-
Rigid foundation slab (DSD model)	3,69	8,59	27,0%	73,5%
Flexible foundation slab (DSD model)	3,18	8,33	16,5%	74,2%

For the model with a rigid base, i.e. without SSI, the maximum (relative) displacements and (absolute) accelerations are given in Figure 9a,b. They are plotted at the shear centre of the floor over the building height. The results of the time history analyses (THA) agree well with the response spectrum (RSA) analysis.

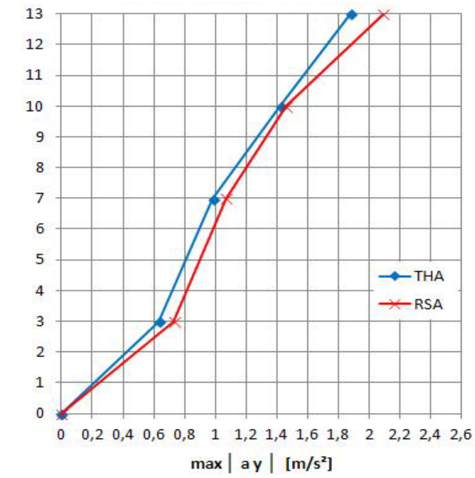
The DSD models have been analysed with and without radiation damping in soil. In the case without radiation damping, a global material damping coefficient of 5% has been assumed in the soil and the structure. Maximum displacements and accelerations are shown in Figure 9c,d. The flexibility of the foundation slab has a remarkable influence on the displacements but only a moderate influence on accelerations. The maximum acceleration at the top of the building with a flexible foundation slab compare well with those of a rigid foundation. The results of the response spectrum analysis are again larger than those of the time history analysis.

In the DSD model with radiation damping the impedance functions are evaluated at the first eigenfrequency of the structure. The damping coefficients acc. to eqn.'s 12a,b for a rigid foundation slab are $\xi_{h,1} = 0{,}58$, $\xi_{h,2} = 1{,}47$, $\xi_{r,1} = 0{,}33$ and $\xi_{r,2} = 1{,}33$ whereas for the flexible foundation slab the damping coefficients are $\xi_{h,1} = 0{,}50$, $\xi_{h,2} = 1{,}41$, $\xi_{r,1} = 0{,}25$ and $\xi_{r,2} = 1{,}26$. This shows that the damping for horizontal motion but also for rocking in the second mode are very high and will prevent these type of vibrations. The weighted modal damping coefficients acc. to eq. 11 are given in Table 2. The accelerations and displacements are considerably reduced by the higher damping. However, the accelerations obtained by a time history analysis with the DSD model for a flexible foundation slab are ~20% larger than those of a SBFEM analysis. For a rigid foundation the differences are only ~10%. This reflects the fact that the simple DSD spring model does not model the interaction between the "individual springs" and thus underestimates the local soil stiffness under a wall. Hence, the best approach with the DSD method is for a rigid foundation slab where the SBFEM and the DSD accelerations differ by about 10% only. The maximum accelerations obtained by a response spectrum analysis are again ~10% larger than those of a time history analysis due to the simplification of the damping model.

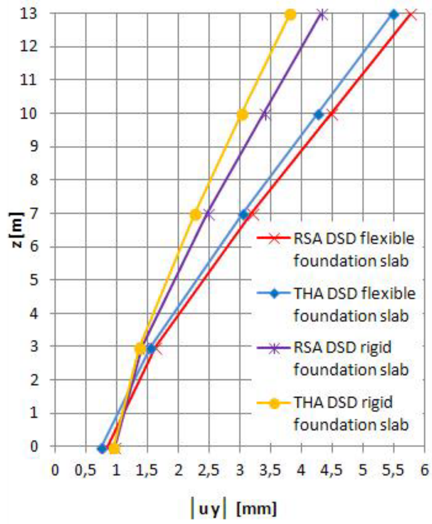
In addition to the results presented here, a beam model of the structure with frequency dependent soil springs and dampers acc. to eq. 1a,b has been computed in frequency domain. The global acclerations agree well with those of the SBFEM model (Volarevic 2013).



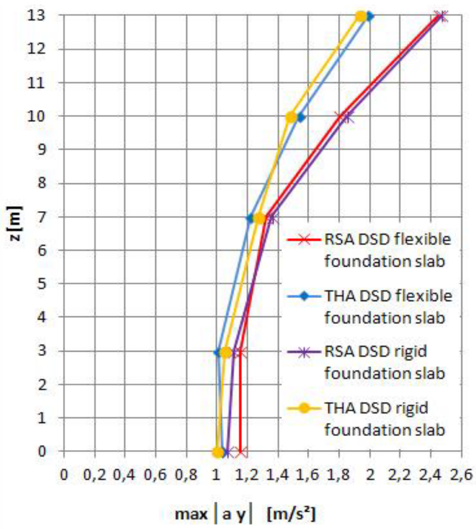
(a) Displacements, model with fixed base



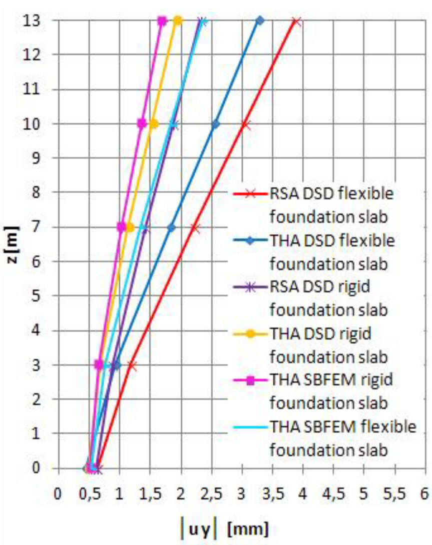
(b) Accelerations, model with fixed base



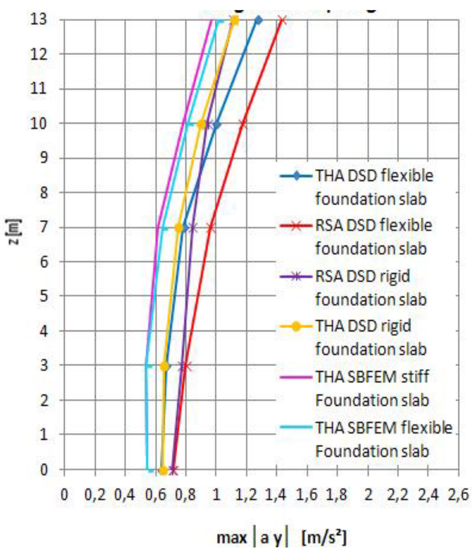
(c) Displacements, model with 5% damping



(d) Accelerations, model with 5% damping



(e) Displacements, model with modal damping

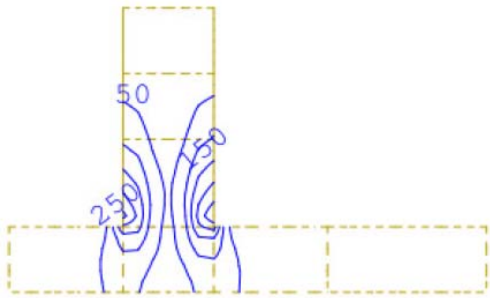


(f) Accelerations, model with modal damping

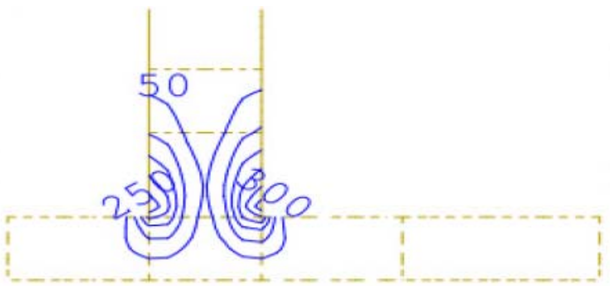
Figure 9. Maximum displacements and accelerations

LOCAL EFFECTS

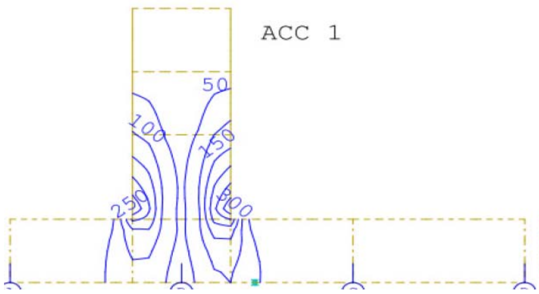
Shell models of buildings give the stresses and stress resultants in all structural elements of the building model. Those are influenced by soil-structure interaction effects. The maximum vertical stresses at the bottom of wall W1 (Figure 5) are taken as example. They are shown in Figure 10 for the time history analyses with the DSD and the SBFEM models. The stresses agree well for the rigid foundation slab but also show some differences for the flexible slab. This indicates some deficiencies of the DSD model to represent the local soil stiffness and interaction of "soil springs" as mentioned before.



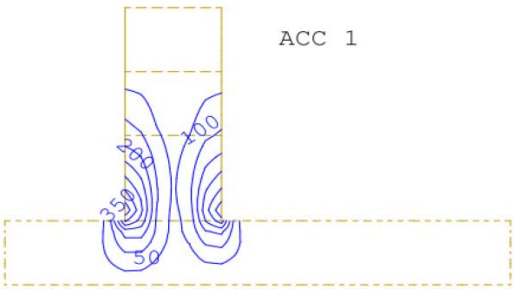
(a) SBFEM – rigid foundation slab



(b) SBFEM – flexible foundation slab



(c) DSD – rigid foundation slab



(d) DSD – flexible foundation slab

Figure 10. Vertical stress resultants [kN/m] at the bottom of wall W1

CONCLUSIONS

Three-dimensional models of buildings are being used more and more frequently due to their benefits in structural analysis. Soil-structure interaction plays an important role in many cases but is not easy to toggle. In this study, the DSD method and the SBFEM have been used. Both methods have advantages but also limitations. The SBFEM method is only available for a homogeneous elastic half-space whereas the impedance functions used in the DSD method can be computed for any arbitrarily layered soil. The local stiffness of the soil is represented well by the SBFEM model, while with some deficiencies by the DSD model. The frequency dependence in the soil-structure interaction covered by the SBFEM is approximated in the DSD model. However, for practical analyses the DSD model is well suited, allowing also a response spectrum analysis if a tolerable overestimation of the structural accelerations is accepted.

The study shows the necessity to continue in the development of soil-structure interaction models and computational methods suitable to model layered and inhomogeneous soil conditions found in nature and at the same time are able to compute those models, being part of a three-dimensional finite element model of a building, efficiently.

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