COMPDYN 2015
5th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, V. Papadopoulos, V. Plevris (eds.)
Crete Island, Greece, 25–27 May 2015

INFLUENCE OF HUMAN INDUCED FLOOR VIBRATIONS ON THE MEASUREMENT PRECISION OF FORCE PLATES IN A BIOMECHANICS LABORATORY

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Keywords: Biomechanics Laboratory, Countermovement Jump, Floor Vibrations, Force Plate, Gait Analysis Laboratory, Human Induced Vibrations.

Abstract. *In biomechanics laboratories the ground reaction force time histories of the footfall of persons are usually measured using a force plate. The accelerations of the floor, in which the force plate is embedded, have to be limited, as they may influence the accuracy of the force measurements. For the numerical simulation of vibrations induced by humans in biomechanical laboratories, loading scenarios are defined. They include continuous motions of persons (walking, running) as well as jumps, typical for biomechanical investigations on athletes. The modeling of floors has to take into account the influence of floor screed in case of portable force plates. Criteria for the assessment of the measuring error provoked by floor vibrations are given. As an example a floor designed to accommodate a force platform in a biomechanical laboratory of the University Hospital in Tübingen, Germany, has been investigated for footfall induced vibrations. The numerical simulation by a finite element analysis has been validated by field measurements. As a result, the measuring error of the force plate installed in the laboratory is obtained for diverse scenarios.*

1 INTRODUCTION

Biomechanics laboratories allow the study and analysis of human motion, especially human gait, by means of the methods of mechanics. They are used in sports science to help athletes to evaluate and improve their performance. Biomechanics or gait analysis laboratories are also used in medicine for orthopaedic purposes such as the analysis of motion restrictions in diagnosis or the verification of orthopaedic rehabilitation measures by means of objective criteria [1] .

The measurements in gait analysis laboratories can be performed by employing different, complementary systems, such as video recordings with subsequent computer analysis, force measurements using force plates, or neurological methods of the electromyography (measurement of the electrical muscle activity). The measurements of the time dependent ground reaction forces of the human footfall are carried out using force plates embedded in the laboratory floor.

Force plates are sensitive to environmental vibrations of the floor which may influence the accuracy of the measurements. The paper investigates human induced vibrations in the laboratory and their influence on the measurement accuracy of a force plate.

2 FORCE PLATES IN GAIT LABORATORIES

The measurement of the force can be done using piezoelectric elements or by the use of strain gauges. In the case of the piezoelectric measuring technique, the piezoelectric effect, i.e. the electrical charge of crystals due to mechanical loads, is used for force measurements. Piezoelectric force plates possess a very high measuring accuracy within an extremely wide measuring range. They will be considered in the following. The reaction force of a multicomponent force plate caused by the footfall load is measured as three-dimensional vector with the vertical and the two horizontal components parallel to the plate edges. Usually a walking and running path parallel to the plate edges is adopted, so that the force components are parallel and perpendicular to the direction of movement, respectively. Fig. 1 shows an example of a measured force time history for a single gait of a walking person with a weight of G = 0.87 kN and a step frequency of f_s = 2 Hz. Herein F_z denotes the vertical force component and F_y, F_x are the horizontal components in the walking direction and perpendicular to it, respectively.

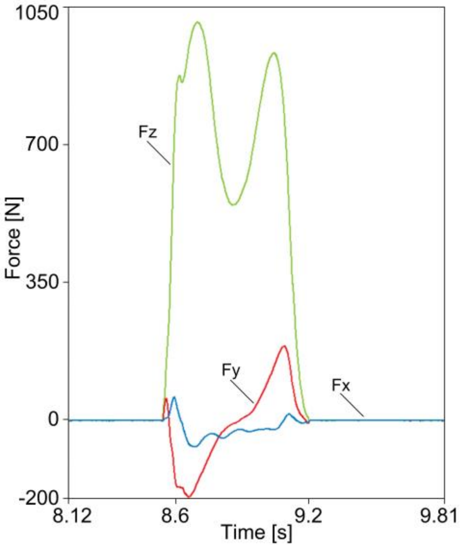

Figure 1: Force time history measured with a force plate (G = 0.87 kN, f_s = 2 Hz).

Force plates consist of a cover plate made of steel, aluminum or glass and the underlying support construction with the piezoelectric sensors (Fig. 2). For a fixed installation in a laboratory, force plates are usually embedded in a planned indentation of the floor, using an installation frame. In addition to stationary force plates, portable force plates are also available. They are mobile and can be used on any flat surface (Fig. 3). Portable force plates allow force measurements in various locations as e.g. in field situations where no mounted force plates are available.

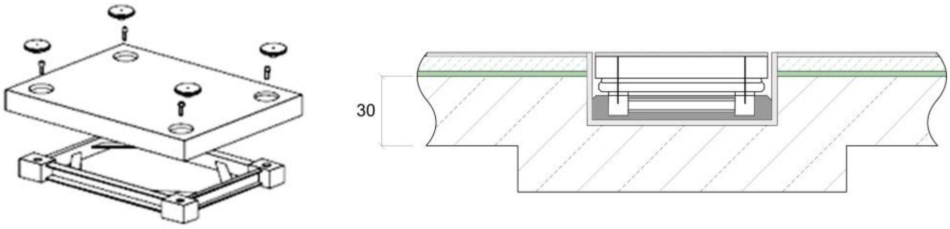

Figure 2: Force plate and installation frame in a floor indentation

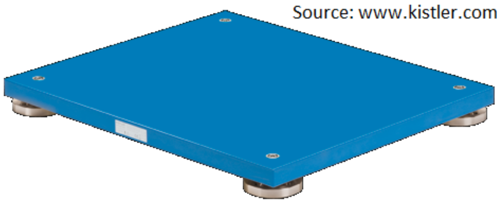
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Figure 3: Portable force plate

3 MEASURING ACCURACY OF FORCE PLATES

Floor vibrations can be induced by the person being subject of the force measurement or by other persons moving around in the laboratory [2, 3]. Other external sources of vibration may be machinery vibrations, traffic induced vibrations or construction-related vibrations. Accelerations of the floor, in which a force plate is embedded or on which a portable plate is deployed may alter the measuring signal. Especially piezoelectric force plates are able to monitor extremely small forces indicating floor accelerations without any loading. In order to limit the resulting measuring error, the floor accelerations have to be limited. In the following the measuring error allowed is assumed to be 1% of the maximal force measured.

The vertical eigenfrequency of the force plates is usually very high (> 500 Hz) due to the very stiff bearing elements. Therefore internal vibrations of the plate device as well as vibrations of the plate relative to the floor can be excluded, presuming that the plate was installed properly. Hence, the relative measuring error p can be determined as a ratio of the mass force of the cover plate and the maximum force measured as

$$p = \frac{m_{Pl} \cdot a_g}{F_{\max}} \tag{1}$$

where m_{Pl} is the mass of the cover plate, a_g is the acceleration of the floor and $F_{\max}$ is the maximum force measured [4]. The limit value of the floor acceleration for a given measuring accuracy is determined to be

$$a_g = \frac{F_{\max} \cdot p}{m_{Pl}} \tag{2}$$

Assuming a tolerable measuring error of 1% and a static load due to a subject with a weight force of 0.4 kN and mass of the cover plate of 40 kg, a threshold value of the acceleration of 0.1 m/s² is obtained. This value is given in some specifications of force plates [4]. However, it should be noted that this simple threshold value of 0.1 m/s² neglects the influence of the weight of the individual, dynamic effects of the force time history and the weight of cover plate of the specific force plate device.

The force to be measured is proportional to the weight of the subject i.e. $F_{\max} = \overline{\alpha} \cdot G$. If the coefficient $\overline{\alpha}$ is not determined by force time history measurements, lower limits are to be chosen. The following values for $\overline{\alpha}$ are adopted: 1.0 for static load (standing), 1.1-1.2 for walking [5], 1.8-2.2 for running and 3.0 for jumping [6]. Hence the relative measuring error for an external excitation inducing the floor acceleration a_{g0} is

$$p = \frac{m_{Pl} \cdot a_{g0}}{\overline{\alpha} \cdot G} \tag{3}$$

In the case of vibrations induced by the person being subject to the measurement, the maximum acceleration of the floor is proportional to the weight G of the subject. Employing the acceleration normalized to the subject weight, $\overline{a} = a_g / G$, the relative error of the measurement is determined to be

$$p = \frac{m_{Pl} \cdot \overline{a}}{\overline{\alpha}} \tag{4}$$

Both types of excitations are to be superposed [7]. Assuming a random distribution of the time instants at which the maxima of both time histories occur, the measuring error is determined to be

$$p = \frac{m_{Pl}}{\overline{\alpha}} \cdot \sqrt{\left(\frac{a_{g0}}{G}\right)^2 + \overline{a}^2} \tag{5}$$

4 LOADING SCENARIOS

The loading scenarios are intended to describe typical situations during force measurements in orthopedic and sport biomechanics laboratories. In the following, only human induced vibrations are considered. If external sources of vibrations are existent, they must be examined in addition.

In either case the vibrations induced by the person being subject to the force measurement must be investigated. The subject may be walking or running on a runway in the gait laboratory. In sport science different types of jumps are typical for tests on a force plate [8]. In addition vibrations may be induced by persons moving around in the laboratory during the tests. The following typical scenarios can be assumed:

- Walking: The subject is walking on a predefined path (runway), including the force plate.
- Running: The subject is running on a predefined path (runway), including the force plate.
- Counter movement jump: The subject stands on the force plate with straight legs, and performs a jump starting with bending the knees in a quick counter movement.
- Squat jump: The subject stands on the force plate, squats down until the knees are bent about 90 degrees and then performs a jump.

- Drop jump: The subject jumps from an elevated platform and performs a jump similar to the counter movement jump after landing.
- Repetitive hops: The subject stands on the force plate and performs a series of jumps.
- Persons moving around in the laboratory: 5 people with a weight of 80 kg each are walking around in the room. The walking path is random.

Depending on the function of the laboratory, the scenarios relevant for the analysis are to be defined in the individual case.

5 STRUCTURAL ANALYSIS

Footfall induced vibrations can be computed with a time history analysis of a finite element model. As reinforced concrete floors typically possess eigenfrequencies above 7-10 Hz, simplified approaches for “high-frequency floors” representing the load by equivalent impulses may be applied. Both methods are addressed in the following.

5.1 Load Models

The load generated by a single walking or running subject can be described by load functions for every step or simplified, using a continuous, time dependent, single load. Zivanovic [9], Butz [10], Werkle [11] and Racic [12] give overviews over various approaches for human-induced walking and running loads.

In the following, the time dependent load is represented as a Fourier series:

$$F(t) = G \cdot \left(1 + \sum \alpha_j \cdot \sin\left(2 \cdot \pi \cdot j \cdot t \cdot f_s - \varphi_j\right)\right) \tag{6}$$

Here f_s is the step frequency. The subject weight was assumed to be 0.8 kN. The Fourier-coefficients α and φ have been determined by different authors. Table 1 shows the values for walking and running according to Bachmann [13] and higher frequency terms estimated based on tentative tests in [14]. These higher Fourier coefficients include the heel-strike effect and are important for the vibration response of high-frequency floors [15, 16].

The load is moving along a predefined path on the floor with the speed

$$v = f_S \cdot L_S \tag{6a}$$

where f_s denotes the step frequency which is to be chosen conservatively and L_S represents the step length. Typical values are given by [9-11].

		Walking	**Running**
j	$\mathbf{f_s}$	1.5 – 2.5 [Hz] (Average value 2 Hz)	2.0 –3.0 [Hz]
1	α_1	0.4 for $f_s \le 2$ *Hz* $0.4 + 0.1 \cdot (f_s - 2) / 0.4$ for 2.0 *Hz* $\le f_s \le$ 2.4 *Hz* 0.5 for $f_s \ge 2.4$ *Hz*	1.6
2	α_2	0.1	0.7
3	α_3	0.1	0.2
4	α_4	0.07*	0.15*
5	α_5	0.06*	0.12*
6	α_6	0.06*	0.12*
1	φ_1	0	0
2	φ_2	$\pi / 2$	$\pi / 2$
3	φ_3	$\pi / 2$	$\pi / 2$

Table 1: Coefficients α and φ for walking and running ([13] and * acc. to [14])

Eq. 6 is valid for a single person. For an excitation induced by several persons, the acceleration obtained for “walking” of a single person is multiplied by the factor $m = \sqrt{n}$, where n is the number of the persons.

High-frequency floors may also be analyzed by an effective impulse approach as given in [16, 17]. The method is based on the assumption that due to the high frequency response and the damping, the transient vibration generated by an individual footfall practically has vanished before the next footfall begins.

Equivalent impulses for a countermovement jump and a drop jump have been developed by Firus et. al [18] based on force time history measurements of more than 200 jumps. For a countermovement jump the equivalent impulses in kNs are given by

$$I_{50\%} = G \cdot 0{,}4028 \cdot f^{-0{,}810} \quad (p = 50\%)\,, \tag{7a}$$

$$I_{25\%} = G \cdot 0{,}3471 \cdot f^{-0{,}642} \quad (p = 25\%)\,, \tag{7b}$$

where *f* denotes the corresponding eigenfrequency of the structure in [Hz], *G* represents the subject weight in [kN] and *p* = 50% and *p* = 25% are the exceedance probabilities, respectively. The paper also gives a procedure for the analysis of drop jumps using equivalent impulses [18].

5.2 Structural Model

The structural finite element model must be detailed with care in order to represent the response of the structure adequately. For continuous slabs the number of floor panels considered in the structural model must be large enough to include all mode shapes relevant for the vibration response. For a stationary force plate (Fig. 2) the modelling of the raw concrete floor is appropriate. However, if a portable force plate (Fig. 3) is used, the structural model should include a floating floor screed which may affect the local acceleration significantly.

5.3 Time history analyses

Time history analyses can be performed by direct integration or by modal superposition. In a modal superposition approach, the number of mode shapes to be considered in the analysis must be chosen carefully. It should not only reflect the frequency content of the load time history but also enable all relevant static deformations of the model. In order to describe local deformations on the top of a floating floor screed and the corresponding accelerations, higher mode shapes must necessarily be included in the analysis. This applies for modal superposition but it is not relevant for direct integration.

5.4 Equivalent impulse analyses

The structural response caused by an impulse can be easily computed analytically by modal superposition. The acceleration *a(t)* is obtained as:

$$a(t) = \mu_i \cdot \mu_j \cdot \frac{I}{m_{\text{mod}}} \cdot e^{-\xi \cdot \omega \cdot t} \cdot \left(-\omega_d \cdot \sin(\omega_d \cdot t) - \xi \cdot \omega \cdot \cos(\omega_d \cdot t)\right) \tag{8}$$

where $I\ [kN \cdot s]$ is the impulse, μ_j the mode shape at the point where the impulse is applied, μ_i the mode shape at the point where the response is measured (i.e. where the force plate is situated), $m_{\text{mod}}\ [to]$ the modal mass (using the same scaling of the mode shapes as for μ_i and μ_j), ξ the damping ratio and $\omega = 2 \cdot \pi \cdot f\ [Hz]$ and $\omega_d = \omega \cdot \sqrt{1-\xi^2}$ the undamped and damped angular eigenfrequency of the floor, respectively. In order to consider several modes, the responses of the individual mode shapes have to be superposed.

Figure 4: Health Center Tübingen: Building and biomechanics laboratory

6 EXAMPLE

6.1 Building

The influence of floor vibrations on the measurement accuracy of a force plate has been investigated for a biomechanics laboratory in Tübingen, Germany. The laboratory is located in the new building of the Health Center at the University Hospital in Tübingen, which was built between 2011 and 2012 (Fig. 4). The gait analysis facilities are situated in an open space area, used for office as well as for therapeutic purposes. The paper describes the investigation of the influence of floor vibrations, induced by footfall in the office section, on the accuracy of the simultaneous measurements with the force plate. For this, vibration field measurements and finite element analyses were made.

6.2 Structural model

The floor where the biomechanics laboratory is situated consists of a reinforced concrete slab with a thickness of 30 cm and span widths of the floor panels between 8 and 11 m. The entire floor is represented by a detailed finite element model which consists of 6615 quadrangular plate-elements, containing 40725 degrees of freedom. The reinforced concrete floor is assumed to be elastically clamped by the outer walls. A damping ratio of 1% was determined by measurements. The eigenfrequencies of the floor are given in Table 2, some relevant mode shapes are shown in Figure 5.

Eigenmode	FE-Analysis [Hz]	Measured [Hz]	Eigenmode	FE-Analysis [Hz]	Measured [Hz]
1	7.32	7.27	6	13.52	13.68
2	7.68	7.63	7	13.96	13.93
3	8.21	8.07	8	16.19	16.30
4	11.62	11.55	9	17.17	17.32
5	12.87	12.78	10	18.58	18.90

Table 2: Eigenfrequencies of the floor

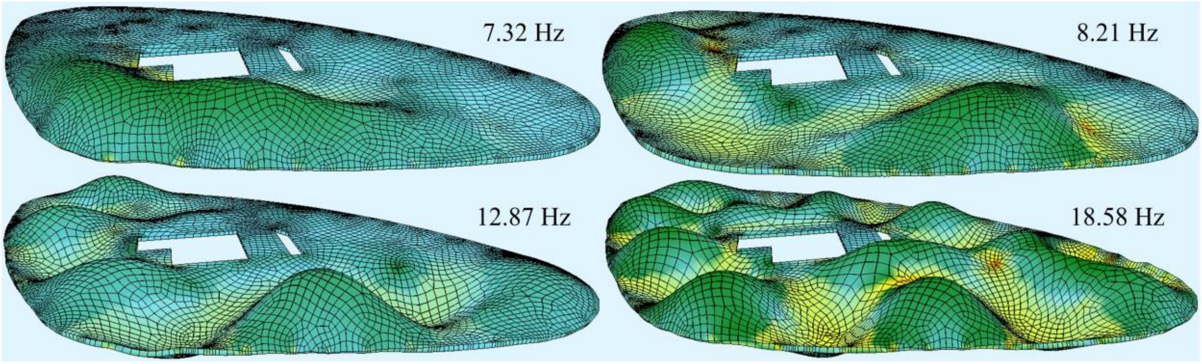

Figure 5: Mode shapes

6.3 Numerical computation of the human-induced vibrations

The time history for a propagating load acc. to eq. (6) was computed by modal analysis. Figure 6 shows typical acceleration time histories of the raw floor at the location point of the force plate. They are shown for the scenarios walking and running on a runway in the laboratory as well as for a countermovement jump acc. to eq. (8). The maximum accelerations for various walking and running frequencies are shown in Figure 9. Eigenfrequencies up to 30 Hz were considered for the computations as well as for the experimental investigations.

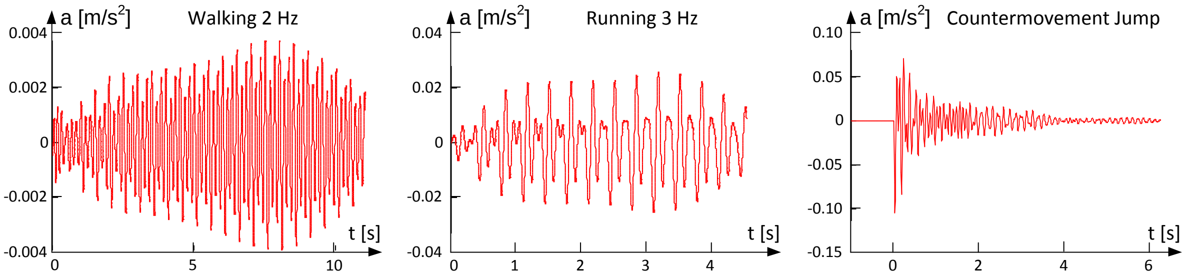

Figure 6: Time histories for walking, running and vertical jumping of a subject (FEM)

6.4 Experimental investigations

The eigenfrequencies of the raw floor were determined through measurements of ambient vibrations and subsequent Fourier analyses. They show a good correlation with the eigenfrequencies obtained by the FE analysis (Table 2).

The vibrations caused by walking, running and jumping of a person were measured using extremely sensitive seismic accelerometers. The subjects wore sports shoes, which are typical for the activities in the biomechanics laboratory. For each scenario, acceleration time histories at different relevant points were recorded. The results were normalized to a subject weight of 0.8 kN.

These tests were done for two stages of the construction, namely the raw concrete floor and the final state of the finished floor, including a layer of sound insulation and a floating floor screed. Figures 7 and 8 show the measured time histories for walking and running on the raw concrete floor and on the finished floor, respectively.

On the raw concrete floor, the maximum accelerations agree well with those of the finite element analysis. Additional investigations showed that walking and running on the finished floor did not significantly affect the accelerations on the raw floor. However the accelerations measured on top of the finished floor are significantly larger than those on the raw floor. It can be noted that the maximum acceleration of the floor in the final construction stage is represented by a single peak at the moment when the subject is stepping close to the acceleration sensor. This indicates that the local deformation of the soft sound insulation layer of the finished floor and the accelerations resulting thereof are the cause of the local augmentation of the structural response.

Hence, for a stationary force plate embedded in the concrete slab, computations can be made for a raw floor neglecting the finishing of the floor. However, for portable force plates which are placed on the finished floor, the additional accelerations caused by the finishing of the floor should be taken into account.

Figure 9 shows the maximum accelerations for walking and running at different step frequencies measured in both construction stages, as well as for the finite element analysis of the raw floor.

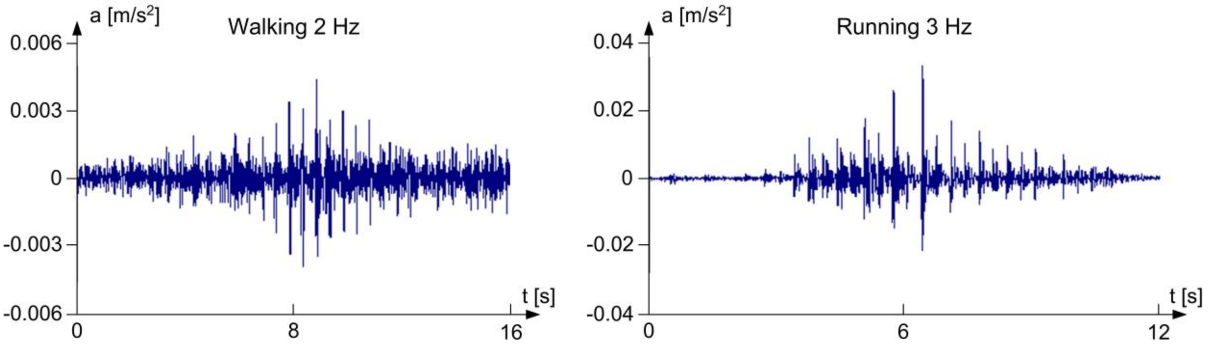

Figure 7: Measured time histories for walking and running (raw concrete floor)

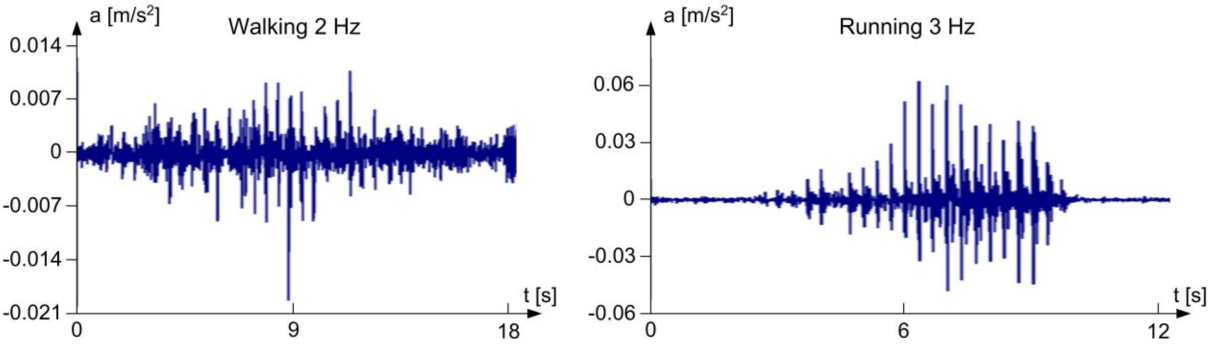

Figure 8: Measured time histories for walking and running (finished floor)

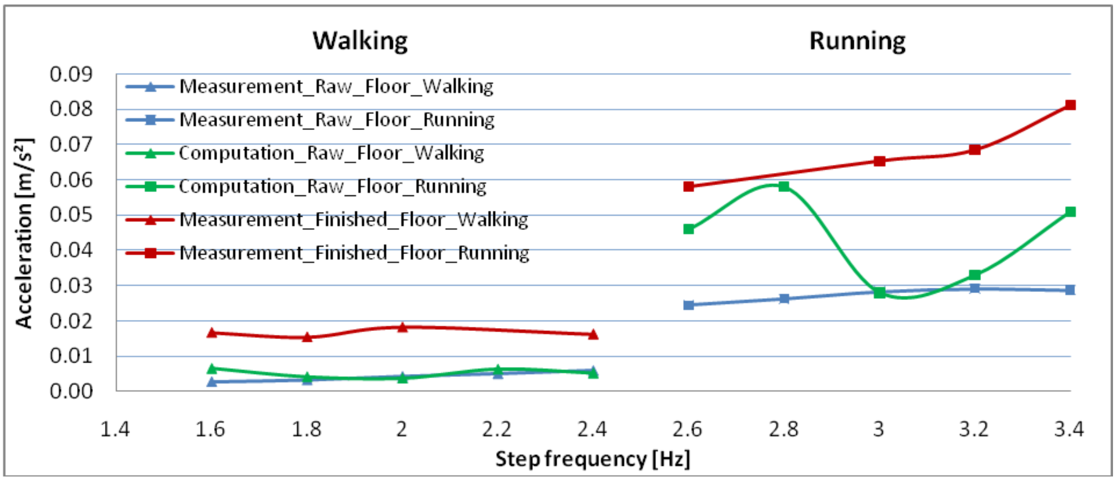

Figure 9: Maximum accelerations of the force plate – computation and measurements

A good correlation between the measurements performed on the raw concrete floor and the corresponding computations can be observed if some resonance effects computationally obtained for running are neglected. However it should be noted that the accelerations of 0.005 m/s^2 for walking and 0.03-0.06 m/s^2 for running are quite low. The highest accelerations in this stage of construction, of about 0.1 m/s^2, occur as a response to the countermovement jump.

The measurements in the final stage of the finished floor, however, revealed much higher accelerations than the ones of the raw concrete floor for the same scenarios investigated. The maximum accelerations in this case are about 2 to 4 times higher than the accelerations of the raw floor. The maximum value of 0.25 m/s^2 was obtained again for jumping.

6.5 Measurement accuracy of the force plate

The measurement accuracy has been evaluated for a stationary plate of the type Kistler 9287 CA. Its total mass is 25 kg, the cover plate has a mass of 20,7 kg [4]. The weight of the subject was assumed to be 0.8 kN, while the values of the coefficient $\bar{\alpha}$ are taken to 1 for walking, 2 for running and 3 for vertical jumping. Furthermore it is assumed that during the measurement 5 persons are walking in the laboratory room. The excitation by the person being subject to the force measurement as well as the excitation by the persons walking in the room is considered to act simultaneously.

The measuring error of the force plate was determined according to Eq. 5 employing the measured floor accelerations. For all scenarios considered, the measuring error was calculated to be less than 0.1% (see Table 3). Even when parameter variations due to model uncertainties (induced, for instance, by the shoe type) are considered, the measuring error remains in the per mille domain and thereby very low.

If a portable force measuring plate with the same mass will be used in the laboratory, it will be placed on the finished floor, which possesses higher floor accelerations than the raw concrete floor. Therefore the measuring error increases significantly, up to 0.2% for the worst case scenario of a subject performing a jump with simultaneous external excitation by persons walking in the laboratory (see Table 4).

It should be noted that the measuring error of the force plate would be slightly higher for a more conservative assumption of the subject weight, e.g. 0.5 kN.

Scenario	**Floor acceleration (1 person) [m/s²]**	**Absolute error [N]**	**Maximal force assumed [kN]**	**Relative error induced by subject (Eq. 4)**	**Total error including external excitation (Eq. 5)**
Walking	0.007 (0.016*)	0.1	0.8	0.02 %	0.03 %
Running	0.058**	1.0	1.6	0.07 %	0.07 %
Countermovement Jump	0.115**	2.4	2.4	0.10 %	0.10 %

Table 3: Measuring error – stationary force plate (* for 5 persons; **computed values)

Scenario	**Floor acceleration (1 person) [m/s²]**	**Absolute error [N]**	**Maximal force assumed [kN]**	**Relative error induced by subject (Eq. 4)**	**Total error including external excitation (Eq. 5)**
Walking	0.02 (0.045*)	0.4	0.8	0.05 %	0.13 %
Running	0.081	1.7	1.6	0.11 %	0.12 %
Countermovement Jump	0.25	5.2	2.4	0.22 %	0.22 %

Table 4: Measuring error – portable force plate (* for 5 persons)

7 CONCLUSIONS

The measuring error of force plates in biomechanics laboratories can be evaluated based on experimental and numerical investigations. Scenarios and formulas for determining the measuring error were proposed. The largest accelerations are obtained for jumps being part of typical investigations in sports gait analysis laboratories. Portable force plates are subjected to larger floor accelerations than stationary ones due to the augmentation of vibrations by the floor screed and sound insulation.

It was shown that the measuring accuracy of the force plate from the biomechanics laboratory at the University Hospital in Tübingen, Germany, is not significantly influenced by the human induced vibrations of the floor. This may be typical for similar floors consisting of a thick reinforced concrete slab. In cases of steel constructions or composite slabs, larger accelerations which may influence the measuring accuracy of the force plate noticeably are to be expected.

ACKNOWLEDGEMENTS

The authors would like to thank Prof. Dr. Stefan Grau for supporting the experimental investigations as well as for providing measurement data of the force plate.

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